Fuzzy logic:

The fundamentals of fuzzy logic elaborated by LotfiA.Zedeh, a professor at the University of California at Berkley.He presented fuzzy logic not as a control methodology, but as a method of processing data by allowing partial set membership instead of non membership. Until 70's due to insufficient small computer capability the method of settheory was not applied to control system.
Nonlinear mapping of an input data set to a scalar output data is known as fuzzy logic system.

A fuzzy logic system consists of four main parts:
· Fuzzifier
· Rules
· inference engine
· defuzzifier.
These components and the general architecture of a fuzzy logic system are shown in Figure

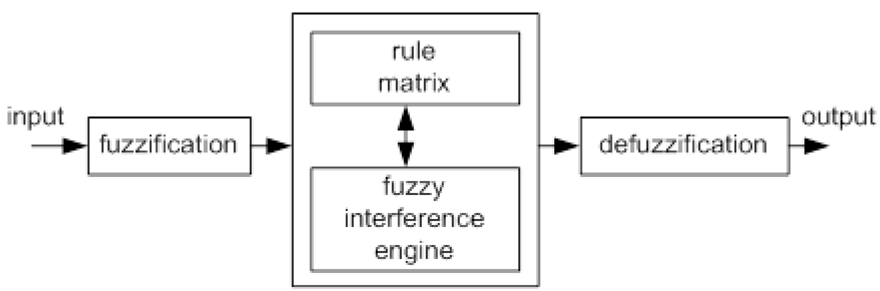


Figure 2. Fuzzy logic controller [1]

These complex systems can be simplified by employing a tolerance margin for a reasonable amount of imprecision, vagueness and uncertainty during the modelling phase. As an outcome, not completely perfect system comes to existence; nevertheless in most of the cases it is capable of solving the problem in appropriate way. Even missing input information has already turned out to be satisfactory in knowledge-based systems.

Fuzzy logic allows to lower complexity by allowing the use of imperfect information in sensible way. It can be implemented in hardware, software, or a combination of both.

In other words, fuzzy logic approach to problems' control mimics how a person would make decisions, only much faster.

The fuzzy logic analysis and control methods shown in Figure 1 can be described as:

1. Receiving one or large number of measurements or other assessment of conditions existing in some system that will be analysed or controlled.
2. Processing all received inputs according to human based, fuzzy "if-then" rules, which can be expressed in simple language words, and combined with traditional non-fuzzy processing.
3. Averaging and weighting the results from all the individual rules into one single output decision or signal which decides what to do or tells a controlled system what to do. The result output signal is a precise defuzzified value.

Below some basic information about fuzzy logic will be presented, while a comprehensive

theory of fuzzy logic can be found in [2].

- **Universe of Discourse**

It is a range of all possible values considered as fuzzy system input.

- **Fuzzy Set**

A fuzzy set *μ* is a function from the reference set *X* to the unit interval, i.e.

$$\mu : X \rightarrow [0,1]$$

μ(X) represents the set of all fuzzy sets of *X*.

- **Membership Function**

It is a graphical representation of fuzzy sets, $\mu_F(x)$.

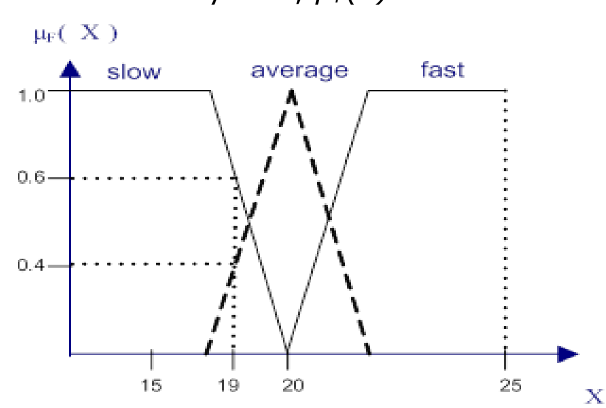


Figure 3. An example of fuzzy logic membership function

Figure 3 shows the membership functions of three fuzzy sets, "slow", "average", and "fast", for a fuzzy variable Velocity. The universe of discourse creates all possible values of Velocity, i.e., X = 19. For Velocity value 19 km/h, the fuzzy set "slow" has the membership value 0.6. Hence, μslow(19) = 0.6. Similarly, μaverage(19) = 0.4, and μfast(19) = 0.

- **Support**

The support of a fuzzy set *F* is the crisp set of all points in the Universe of Discourse *U* such that membership function of *F* does not equal zero

$$\mu_F(u) > 0 \qquad (2)$$

- **Crossover point**

It is an element in *U* where its membership function equals 0.5.

- **Centre**

The centre of a fuzzy set *F* is the point (or points) at which $\mu_F(u)$ achieves its maximum value.

Fuzzification Method

First phase of fuzzy logic proceeding is to deliver input parameters for given fuzzy system based on which the output result will be calculated. These parameters are fuzzified with use of pre-defined input membership functions, which can have different shapes. The most common are: triangular shape, however bell, trapezoidal, sinusoidal and exponential can be also used. Simpler functions will not require complex computing and will not overload the implementation. The degree of membership function is determined by placing a chosen input variable on the horizontal axis, while vertical axis shows quantification of grade of membership of the input variable. The only condition a membership function must meet is that it must vary between zero and one. The value zero means that input variable is not a member of the fuzzy set, while the value one means that input variable is fully a member of the fuzzy set.

With each input parameter there is a unique membership function associated. The membership functions associate a weighting factor with values of each input and the effective rules. These weighting factors determine the degree of influence or degree of membership (DOM) each active rule has. By computing the logical product of the membership weights for each active rule, a set of fuzzy output response magnitudes are produced. All that remains is to combine and defuzzify these output responses [3].

Rule Matrix

The rule matrix is used to describe fuzzy sets and fuzzy operators in form of conditional statements. A single fuzzy if-then rule can be as follows

If *x* is A then *y* is Z,

where A is a set of conditions that have to be satisfied and Z is a set of consequences that can be inferred.

In rule with multiple parts, fuzzy operators are used to combine more than one input: AND = min, OR = max and NOT = additive complement. Geometrical demonstration of fuzzy operators is shown in Figure 4.

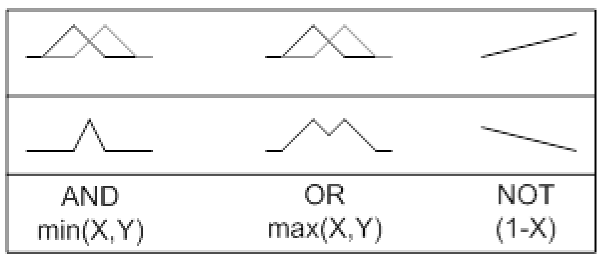


Figure 4. Graphical interpretation of fuzzy operators

The rule matrix is a simple graphical tool for mapping the fuzzy logic control system rules. It accommodates two or more input variables and expresses their logical product (AND or OR) as one output response variable. The degree of membership for rule matrix output can take value of maximum, minimum of the degree of previous of the rule [1]. It is often probable, that after evaluation of all the rules applicable to the input, we get more than one value for the degree of membership. In this case, the simulation has to take into consideration, all three possibilities, the minimum, the maximum or an average of the membership-degrees.

Fuzzy Logic Example

Automotive Speed Controller

3 inputs:

speed (5 levels)

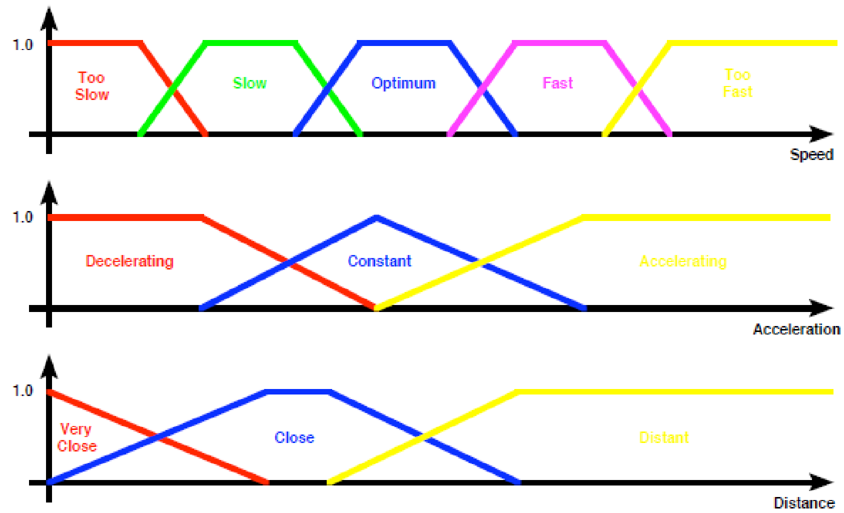
acceleration (3 levels)

distance to destination (3 levels)

1 output:

power (fuel flow to engine)

Set of rules to determine output based on input values



Example Rules

IF speed is TOO SLOW and acceleration is DECELERATING, THEN INCREASE POWER GREATLY

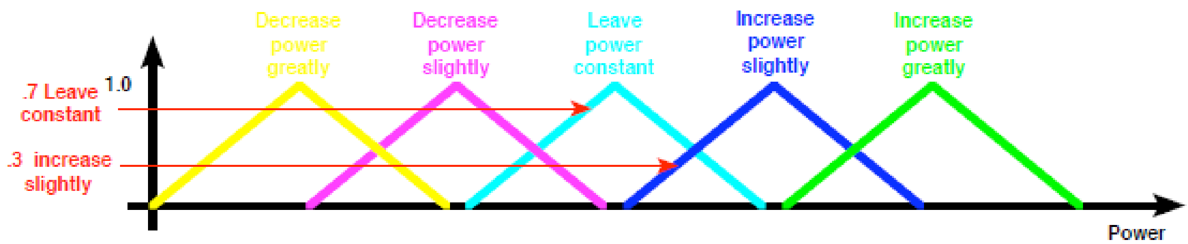
IF speed is SLOW and acceleration is DECREASING, THEN INCREASE POWER SLIGHTLY

IF distance is CLOSE, THEN DECREASE POWER SLIGHTLY

Output Determination

Degree of membership in an output fuzzy set now represents each fuzzy action.

Fuzzy actions are combined to form a system output.


Decrease
power
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.3 increase
slightly
Power