Fuzzy Statistics

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Estimate $P_{1-} P_2$

Chapter 10

10.1 Estimate $p_1 - p_2$, Binomial Populations

We have two binomial population: Pop I and Pop II. Pop I . let p_1 (p_2) be the probability of a "success". We want a fuzzy estimator for $p_1 - p_2 \cdot$ We take a random sample of size n_1 (n_2) from Pop I (II) and observe x_1 (x_2) successes. Then our point estimator for p_1 (p_2) is $\hat{p}_1 = \frac{x_1}{n_1}$ $(\hat{p}_2 = \frac{x_2}{n_2})$, We assume that these two random samples are independent. Then our point estimator of $p_1 - p_2$ is $\hat{p}_1 - \hat{p}_2$.

10.1 Estimate $p_1 - p_2$, Binomial Populations

Now we would like to use the normal approximation to the binomial to construct confidence intervals for $p_1 - p_2$. To do this n_1 and n_2 need to be sufficiently large. So we assume that the sample sizes are sufficiently large so that we may use the normal approximation.

10.1 Estimate $p_1 - p_2$, Binomial Populations

Now $\hat{p}_i$ is (approximately) normally distributed with mean p_i and variance $p_i(1-p_i)/n_i, i = 1,2.$ Then $\hat{p}_1 - \hat{p}_2$ is (approximately) normally distributed with mean $p_1 - p_2$ and variance

$$p_1(1-p_1)/n_1 + p_2(1-p_2)/n_2.$$

This would lead directly to confidence interval, but however we can not evaluate the variance expression because we do not know a value for $p_1 and\ p_2$

10.1 Estimate $p_1 - p_2$, Binomial Populations

We solve this problem by substituting $\hat{p}_i$ for $p_i, i = 1,2$, in the variance equation and use $q_i = 1 - p_i$. Let

$$s_0 = \sqrt{\widehat{p}_1\widehat{q}_1/n_1 + \widehat{p}_2\widehat{q}_2/n_2}.$$

Then

$$P(-z_{\beta/2} \leq \frac{(\widehat{p}_1 - \widehat{p}_2) - (p_1 - p_2)}{s_0} \leq z_{\beta/2}) \approx 1 - \beta.$$

10.1 Estimate $p_1 - p_2$, Binomial Populations

Solve the inequality for $p_1 - p_2$ we obtain an approximate $(1-\beta)100\%$ confidence interval for $p_1 - p_2$ as:

$$[\widehat{p}_1 - \widehat{p}_2 - z_{\beta/2}s_0, \widehat{p}_1 - \widehat{p}_2 + z_{\beta/2}s_0].$$

Put these confidence intervals together to produce our fuzzy estimator $\bar{p}_{12}$ for $p_1 - p_2$.

Example 10.1.1

Let the data be: (1) $X_1 = 63$, $n_1 = 91$; and (2) $X_2 = 42$, $n_2 = 79$. Then the equation becomes

$$[0.1607 - 0.0741 z_{\beta/2}, 0.1607 + 0.0741 z_{\beta/2}].$$

To obtain a graph of $\bar{p}_{12}$ assume that $0.01 \le \beta \le 1$. and then the graph of $\bar{p}_{12}$ is shown in Figure 10.1

Example 10.1.1

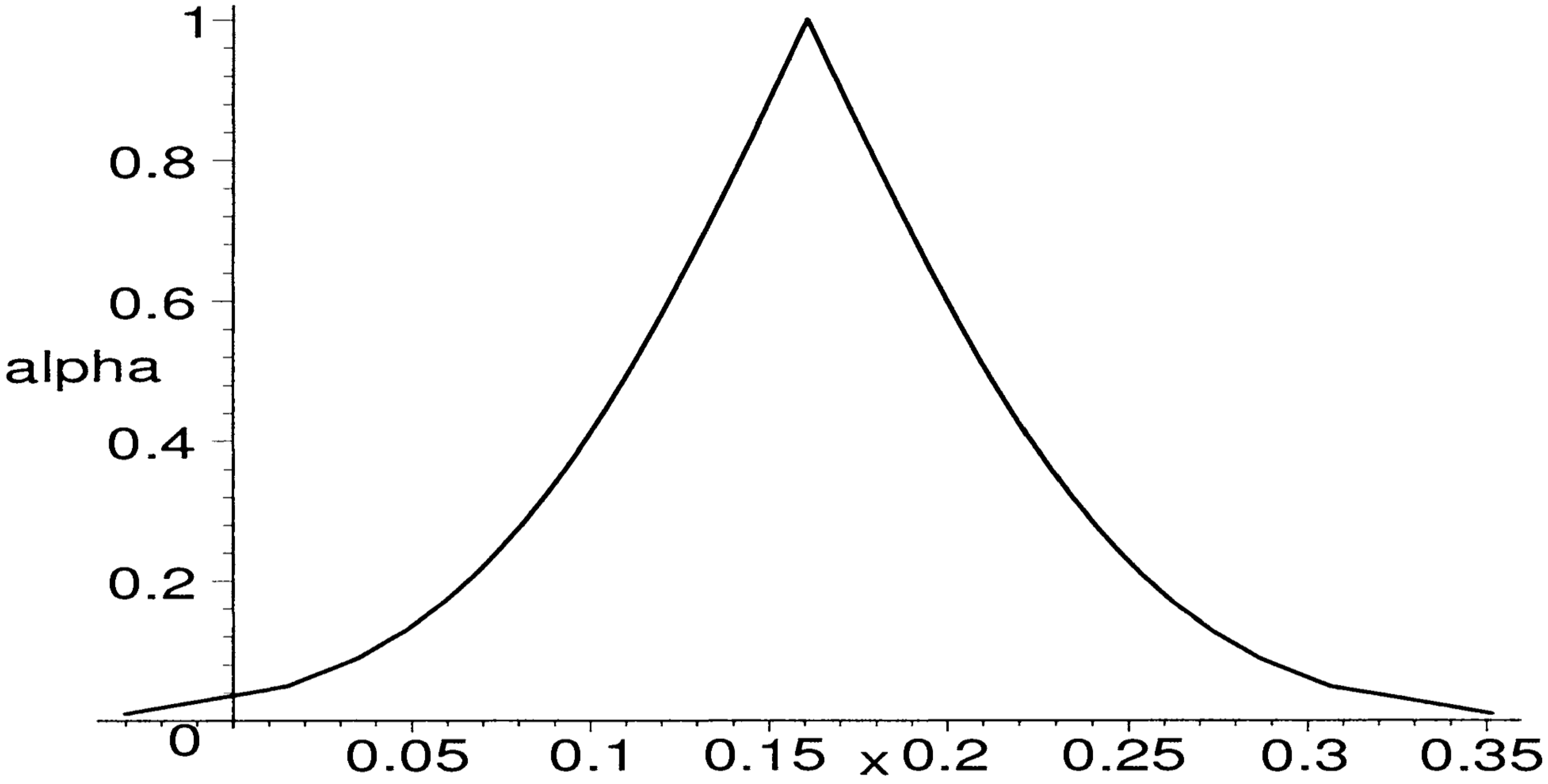

Figure 10.1: Fuzzy Estimator $\overline{p}_{12}$ in Example 10.1.1, $0.01 \leq \beta \leq 1$

Estimate σ_1^2/σ_2^2

Chapter 11

11 Estimate σ_1^2/σ_2^2, Normal Populations

We have two populations: Pop I and Pop II. Pop I is normally distributed with unknown mean μ_1 and unknown variance σ_1^2. Pop II is also normally distributed with unknown mean μ_2 and unknown variance σ_2^2. We wish to construct a fuzzy estimator for σ_1^2/σ_2^2.

11.2 Crisp Estimator

There are two normal populations Pop I and Pop II where: (1) Pop I is N(μ_1,σ_1^2); and (2) Pop II is N(μ_2,σ_2^2). We want to get confidence intervals for σ_1^2/σ_2^2. To estimate σ_1^2 (σ_2^2) we obtain a random sample of size n_1 (n_2) from Pop I (Pop II) and compute s_1^2 (s_2^2) the sample variance. Assume the two random samples were independent. Then we know

$$f_0 = (s_2^2/\sigma_2^2)/(s_1^2/\sigma_1^2).$$

11.2 Crisp Estimator

has a F distribution with n_2 - 1 degrees of freedom (numerator) and n_1 - 1 degrees of freedom (denominator).

$$P(a \le f_0 \le b) = 1 - \beta.$$

Then

$$P(a\frac{s_1^2}{s_2^2} \le \frac{\sigma_1^2}{\sigma_2^2} \le b\frac{s_1^2}{s_2^2}) = 1 - \beta.$$

It immediately follows that a $(1-\beta)100\%$ confidence interval for σ_1^2/σ_2^2 is

$$[a(s_1^2/s_2^2), b(s_1^2/s_2^2)]$$

11.2 Crisp Estimator

Now to determine the a and b.

Assume that X is a random variable from a F distribution with degrees of freedom u (numerator) and v (denominator). Let $F_{L,\beta/2}\,(u,v)$ be a constant so that $P(X \le F_{L,\frac{\beta}{2}}(u,v))$=$\beta/2$ .Also let $F_{L,\beta/2}\,(u,v)$ be another constant so that $P(X \ge F_{L,\beta/2}\,(u,v))$=$\beta/2$. Then the usual confidence interval has a $= F_{L,\beta/2}\,(u,v)$ and b $= F_{L,\beta/2}\,(u,v)$ which gives.

$$[F_{L,\beta/2}(n_2-1,n_1-1)\frac{s_1^2}{s_2^2}, F_{R,\beta/2}(n_2-1,n_1-1)\frac{s_1^2}{s_2^2}], \qquad (11.5)$$

11.2 Crisp Estimator

as the $(1-\beta)100\%$ confidence interval for σ_1^2/σ_2^2. A $(1-\beta)100\%$ confidence interval for σ_1/σ_2 would be

$$[\sqrt{F_{L,\beta/2}(n_2-1,n_1-1)}(s_1/s_2), \sqrt{F_{R,\beta/2}(n_2-1,n_1-1)}(s_1/s_2)].$$

11.3 Fuzzy Estimator

Our fuzzy estimator of σ_1^2/σ_2^2 would be constructed by placing the confidence intervals in equation (11.5) one on top of another. However, this fuzzy estimator is biased. It is biased because the vertex (membership value one) is not at the point estimator S_1^2/S_2^2. To obtain the value at the vertex we substitute one for β and get the 0% confidence interval $[c(s_1^2/s_2^2), c(s_1^2/s_2^2)] = c(s_1^2/s_2^2)$ Where $c = F_{L,0.5}(n_2-1, n_1-1) = F_{R,0.5}(n_2-1, n_1-1).$ Usually the constant $c \neq 1$. We will have c = 1 if $n1 = n2$.

11.3 Fuzzy Estimator

Since c is usually not one the 0% confidence interval will not always be the point estimator. Let us now build an unbiased fuzzy estimator for σ_1^2/σ_2^2.

Our method of making an unbiased fuzzy estimator is similar to what we did in Chapter 6. Assume that $0.01 \le \beta \le 1$. Now this interval for β is fixed and also $n1, n2$, S_1^2 and S_2^2 are fixed. Define

$$L(\lambda) = [1-\lambda]F_{L,0.005}(n_2-1, n_1-1) + \lambda,$$

$$R(\lambda) = [1-\lambda]F_{R,0.005}(n_2-1, n_1-1) + \lambda.$$

11.3 Fuzzy Estimator

The confidence interval for the ratio of the variances is

$$[L(\lambda)\frac{s_1^2}{s_2^2}, R(\lambda)\frac{s_1^2}{s_2^2}],$$

for $0 \leq \lambda \leq 1.$ We start with a 99% confidence interval when $\lambda = 0$ and end up with a 0% confidence interval for $\lambda = 1.$ $L(\lambda)$ $(R(\lambda))$ continuously increases (decreases) to one as λ goes from zero to one. Notice that now the 0% confidence interval is $[s_1^2/s_2^2, s_1^2/s_2^2] = s_1^2/s_2^2$ and it is unbiased. As usual, we place these confidence intervals one on top of another to obtain our (unbiased) fuzzy estimator for $\bar{\sigma}_{12}$ the ratio of the variances.

11.3 Fuzzy Estimator

Our confidence interval for σ_1^2/σ_2^2, the ratio of the population standard deviations, is

$$[\sqrt{L(\lambda)}(s_1/s_2), \sqrt{R(\lambda)}(s_1/s_2)].$$

These confidence intervals will make up our fuzzy estimator $\bar{\sigma}_{12}$ for σ_1^2/σ_2^2. We may find the relationship between λ and β because β is a function of λ given by

$$\beta = \int_0^{L(\lambda)} F dx + \int_{R(\lambda)}^{\infty} F dx,$$

where "F" denotes the F distribution with $n2 - 1$ and $n1 - 1$ degrees of freedom.

Example 11.3.1

From Pop I we have a random sample of size $n1 = 8$ and we compute $s_1^2 = 14.3$. From Pop II the data was $n2 = 12$ and $s_2^2 = 9.8$. Then

$$L(\lambda) = (1 - \lambda)(0.1705) + \lambda,$$

$$R(\lambda) = (1 - \lambda)(8.2697) + \lambda.$$

The confidence intervals become

$$[(0.1705 + 0.8295\lambda)(1.459), (8.2697 - 7.2697\lambda)(1.459)]$$

For $0 \leq \lambda \leq 1.$ the graph of $\bar{\sigma}_{12}$ in Figure 11.1 from above equation.

Example 11.3.1

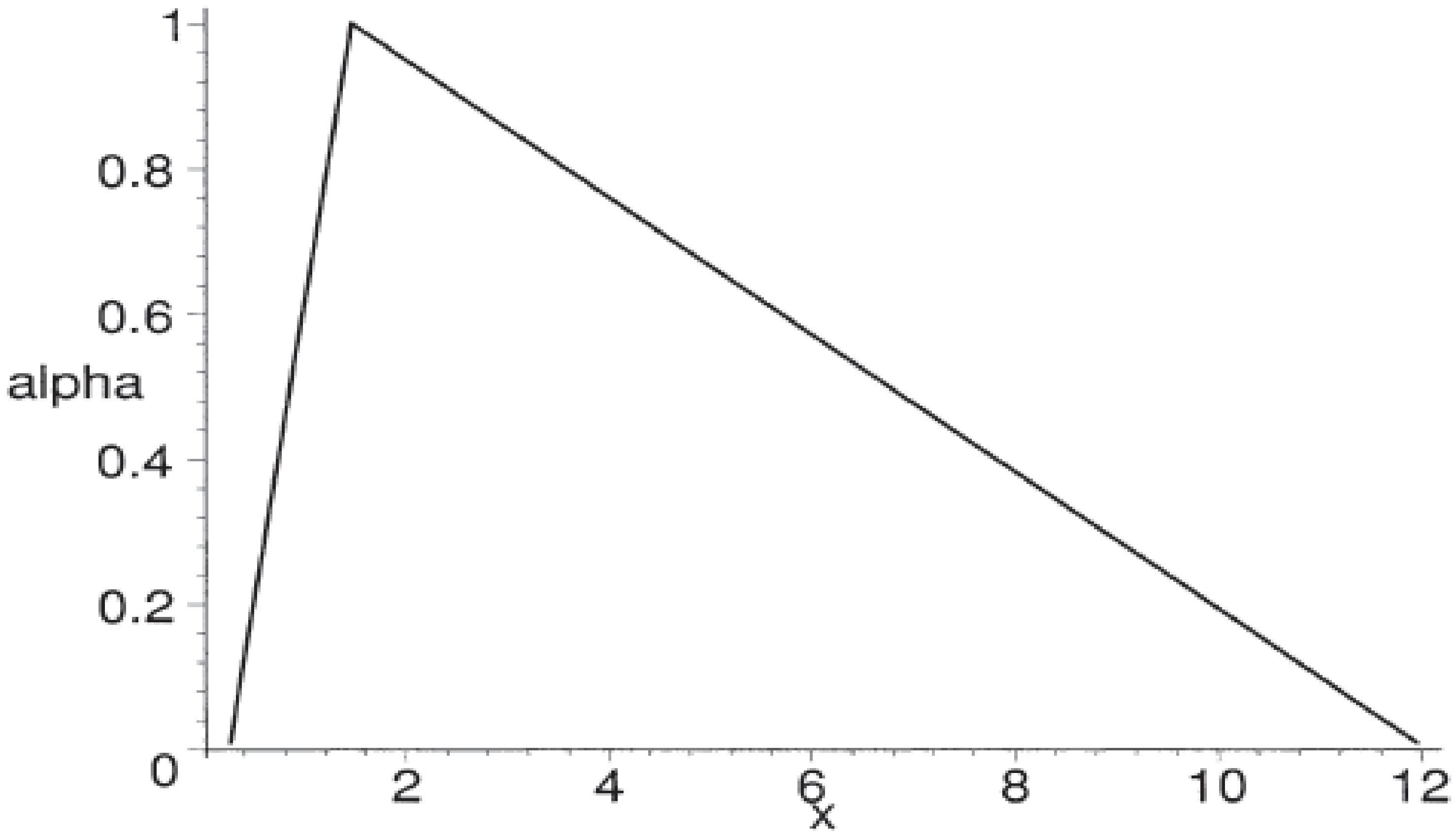

Figure 11.1: Fuzzy Estimator $\overline{\sigma}^2_{12}$ of σ_1^2/σ_2^2 in Example 11.3.1, $0.01 \leq \beta \leq 1$

Example 11.3.1

```
x=linspace(0,15);
y=linspace(0.01,1);
X2L= finv(.995, 11,7);
X2R= finv(0.005,11,7);
f1=((1-y)* X2R + y)*(1.459);
f2=((1-y)* X2L + y)*(1.459);
plot(f1,y,f2,y)
ylabel ('alpha')
xlabel('x')
```