FUZZY STATISTICS

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Fuzzy Number

Chapter 2

2.1 FUZZY NUMBER

A fuzzy number is simply an ordinary number whose precise value is somewhat uncertain. Fuzzy numbers are used in statistics, computer programming, engineering, and experimental science.

If a fuzzy set is convex and normalized, and its membership function is defined in R and piecewise continuous, a fuzzy set A on R must possess at least the following three properties:

(1) A must be a normal fuzzy set;

(ii) ${}^{\alpha}A$ must be a closed interval for every $\alpha \in (0, 1]$;

(iii) The support of A, ${}^{0+}A$, must be bounded.

2.1 FUZZY NUMBER

Fuzzy number is expressed as a fuzzy set defining a fuzzy interval in the real number R. Since the boundary of this interval is ambiguous, the interval is also a fuzzy set. Generally a fuzzy interval is represented by two end points a and b and a peak point c as [a, c , b].

Special cases of fuzzy numbers include ordinary real numbers and intervals of real numbers.

2.1 FUZZY NUMBER

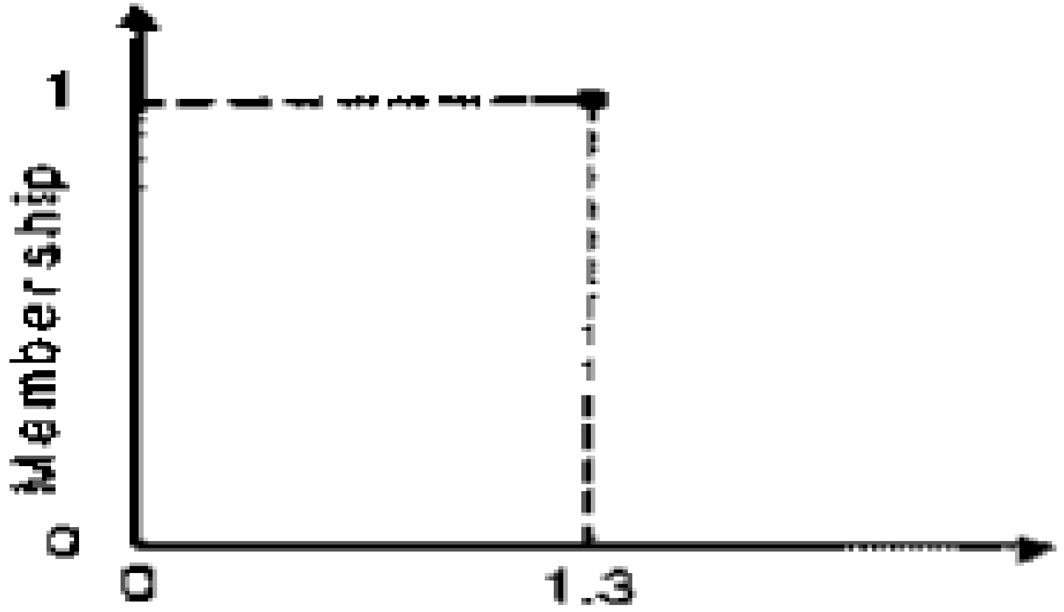

Fig. 2.1: (a) an ordinary real number 1.3

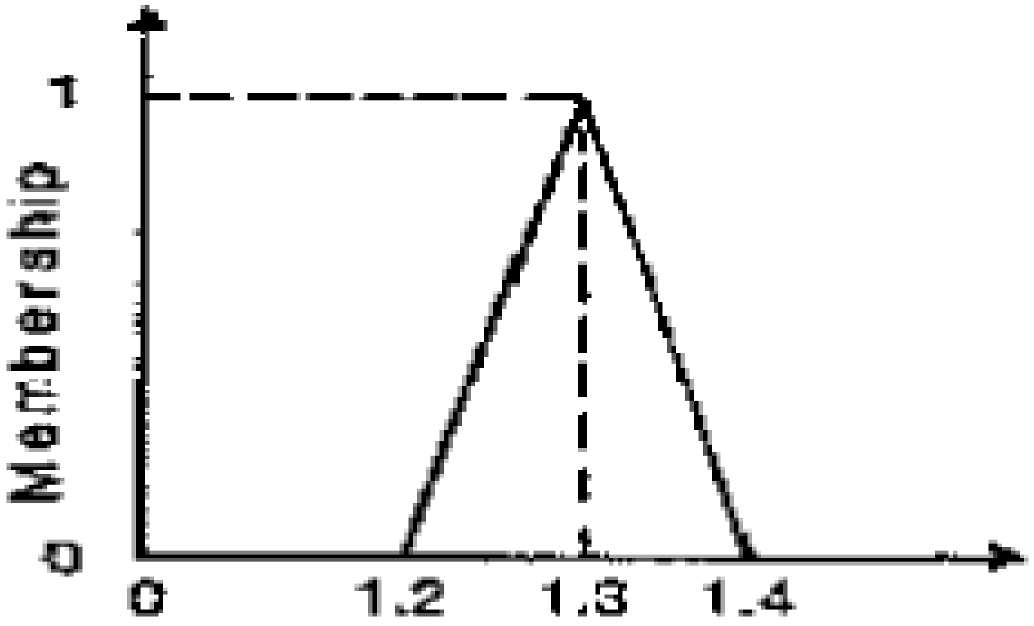

Fig. 2.1: (b) a fuzzy number expressing the proposition "close to 1.3"

2.1 FUZZY NUMBER

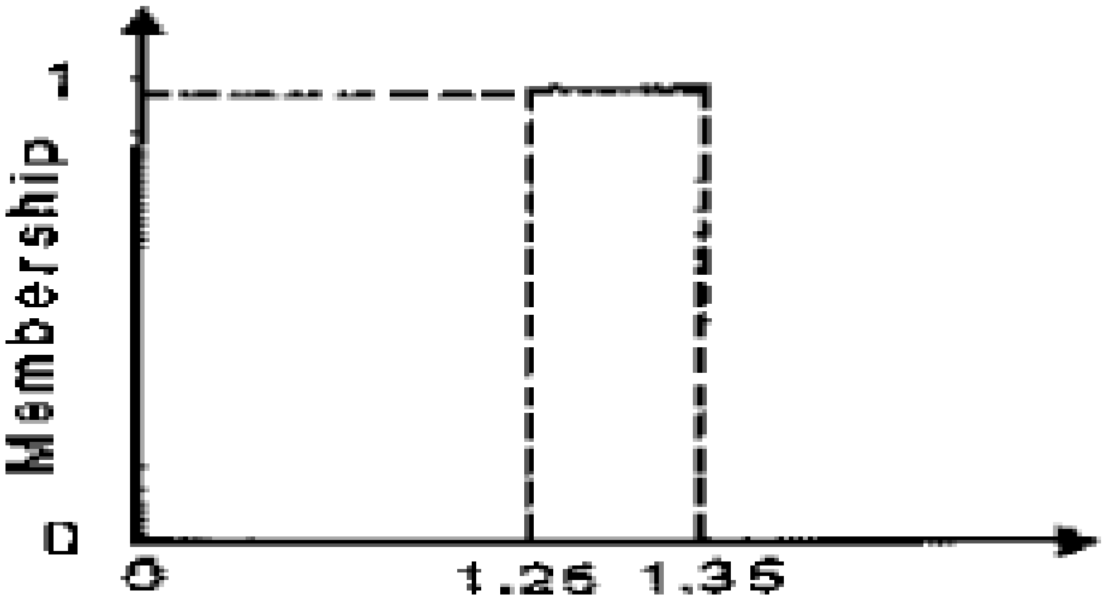

Fig. 2.1: (c) an ordinary (crisp) closed interval [1.25, L35].

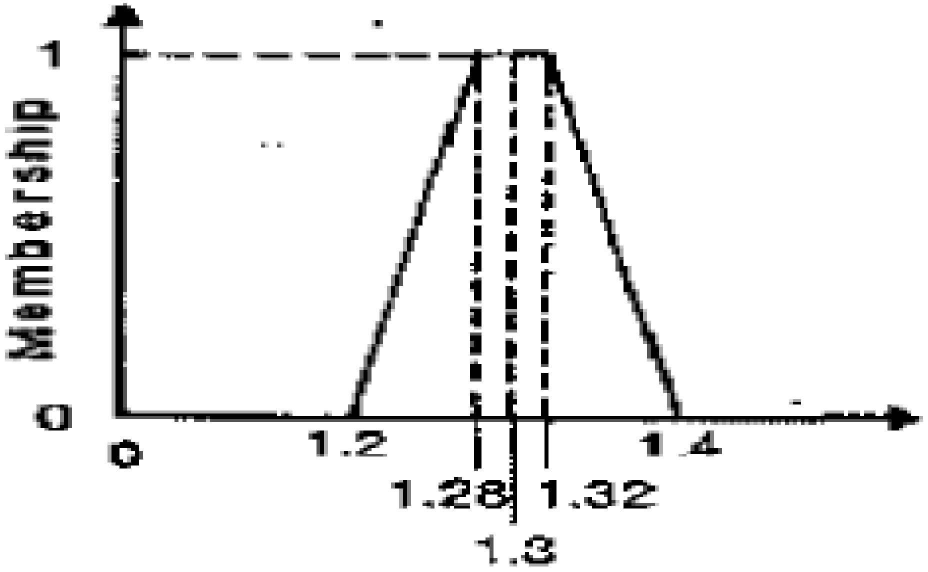

Fig. 2.1: (d) a fuzzy number with a flat region (a fuzzy interval).

2.1 FUZZY NUMBER: TRIANGULAR FUZZY NUMBER

A triangular fuzzy number $\overline{N}$ is defined by three numbers a < b < c where the base of the triangle is the interval [a, c] and its vertex is at x = b. Triangular fuzzy numbers will be written as $\overline{N}$ = (a/b/c). A triangular fuzzy number $\overline{N}$ = (1.2/2/2.4) is shown in Figure 2.2. We see that $\overline{N}(2) = 1$, $\overline{N}(1.6) = 0.5$, etc.

2.1 FUZZY NUMBER

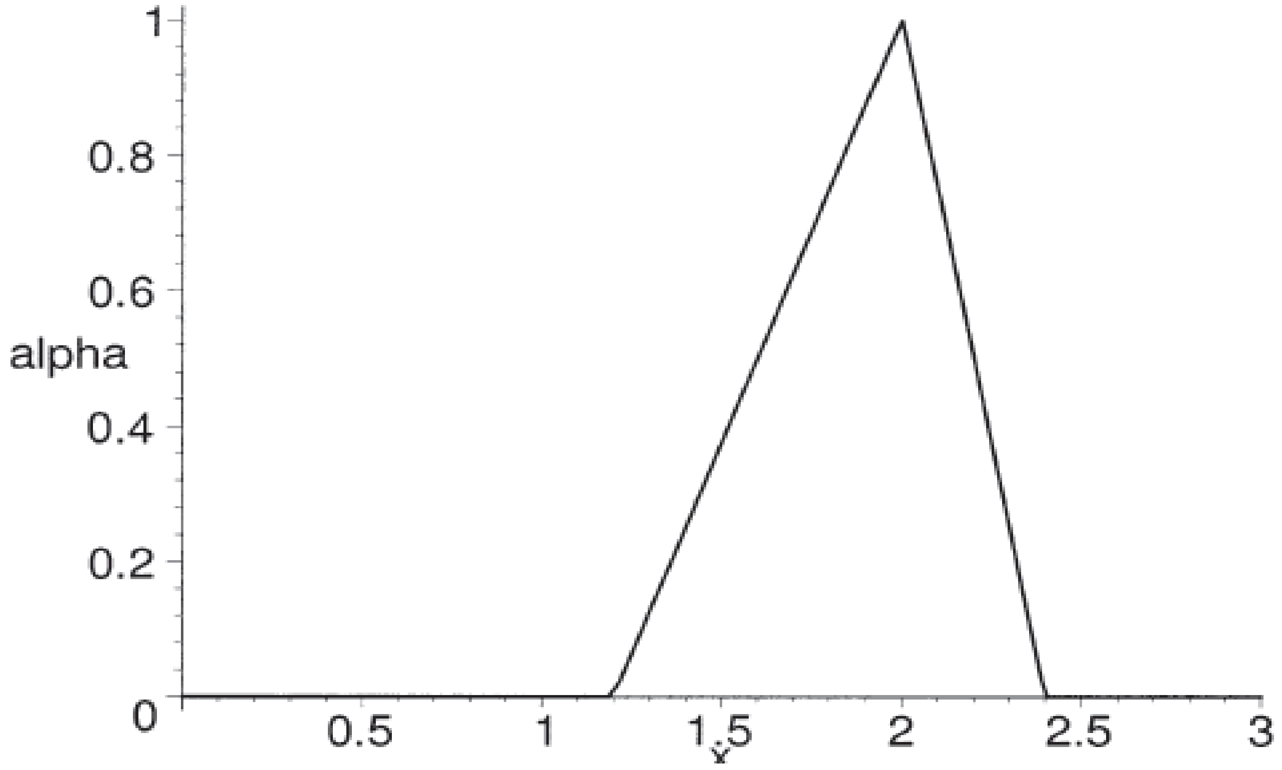

Fig. 2.2: Triangular Fuzzy Number $\overline{N}$

2.2 ARITHMETIC OPERATIONS ON INTERVALS

If $\overline{A}$ and $\overline{B}$ are two fuzzy numbers we will need to add, subtract, multiply and divide them. There are two basic methods of computing $\overline{A}+\overline{B}$, $\overline{A}-\overline{B}$, etc. which are: (1) extension principle; and (2) α-cuts and interval arithmetic.

2.2.1 EXTENSION PRINCIPLE

Let $\overline{A}$ and $\overline{B}$ be two fuzzy numbers. If $\overline{A} + \overline{B} = \overline{C}$, then the membership function for $\overline{C}$ is defined as

$$\overline{C}(z) = \sup_{x,y}\{\min(\overline{A}(x), \overline{B}(y))|x + y = z\} \ .$$

If we set $\overline{C} = \overline{A} - \overline{B}$, then

$$\overline{C}(z) = \sup_{x,y}\{\min(\overline{A}(x), \overline{B}(y))|x - y = z\}$$

2.2.1 EXTENSION PRINCIPLE

Similarly, $\overline{C} = \overline{A} \cdot \overline{B}$, then

$$\overline{C}(z) = \sup_{x,y}\{\min(\overline{A}(x), \overline{B}(y)) | x \cdot y = z\},$$

and if $\overline{C} = \overline{A}/\overline{B}$,

$$\overline{C}(z) = \sup_{x,y}\{\min(\overline{A}(x), \overline{B}(y)) | x/y = z\} \ .$$

2.2.1 EXTENSION PRINCIPLE

In all cases $\overline{C}$ is also a fuzzy number. We assume that zero does not belong to the support of $\overline{B}$ in $\overline{C} = \overline{A}/\overline{B}$. If $\overline{A}$ and $\overline{B}$ are triangular (shaped) fuzzy numbers then so are $\overline{A}+\overline{B}$ and $\overline{A}-\overline{B}$, but $\overline{A}\cdot\overline{B}$ and $\overline{A}/\overline{B}$ will be triangular (shaped) shaped fuzzy numbers.

2.2.2 INTERVAL ARITHMETIC

Let $[a_1,b_1]$ and $[a_2,b_2]$ be two closed , bounded, intervals of real numbers. If $*$ denotes addition, subtraction, multiplication , or division, then $[a_1,b_1] * [a_2,b_2] = [\alpha,\beta]$ where

$$[\alpha,\beta] = \{a * b | a_1 \leq a \leq b_1, a_2 \leq b \leq b_2\} . \tag{2.8}$$

If $*$ is division, we must assume that zero does not belong to $[a_2, b_2]$. We may simplify equation (2.8) as follows:

2.2.2 INTERVAL ARITHMETIC

$$
\begin{aligned}
[a_1, b_1] + [a_2, b_2] &= [a_1 + a_2, b_1 + b_2], \\
[a_1, b_1] - [a_2, b_2] &= [a_1 - b_2, b_1 - a_2], \\
[a_1, b_1] / [a_2, b_2] &= [a_1, b_1] \cdot \left[\frac{1}{b_2}, \frac{1}{a_2}\right],
\end{aligned}
$$

$$[a_1, b_1] \cdot [a_2, b_2] = [\alpha, \beta],$$

2.2.2 INTERVAL ARITHMETIC

where

$$\begin{aligned}\alpha &= \min\{a_1a_2, a_1b_2, b_1a_2, b_1b_2\},\\ \beta &= \max\{a_1a_2, a_1b_2, b_1a_2, b_1b_2\}\ .\end{aligned}$$

2.2.3 FUZZY ARITHMETIC

Again we have two fuzzy numbers $\overline{A}$ and $\overline{B}$. We know α-cuts are closed, bounded, intervals so let $\overline{A}[\alpha] = [a_1(\alpha), a_2(\alpha)]$, $\overline{B}[\alpha] = [b_1(\alpha), b_2(\alpha)]$. Then if $\overline{C} = \overline{A} + \overline{B}$ we have

$$\overline{C}[\alpha] = \overline{A}[\alpha] + \overline{B}[\alpha] \ .$$

We add the intervals using equation (2.9). Setting $\overline{C} = \overline{A} - \overline{B}$ we get

$$\overline{C}[\alpha] = \overline{A}[\alpha] - \overline{B}[\alpha],$$

for all α in $[0, 1]$. Also

$$\overline{C}[\alpha] = \overline{A}[\alpha] \cdot \overline{B}[\alpha],$$

for $\overline{C} = \overline{A} \cdot \overline{B}$ and

$$\overline{C}[\alpha] = \overline{A}[\alpha] / \overline{B}[\alpha],$$

when $\overline{C} = \overline{A}/\overline{B}$, provided that zero does not belong to $\overline{B}[\alpha]$ for all α.

2.2.3 FUZZY ARITHMETIC

EXAMPLE :

Let $\overline{A} = (-3/-2/-1)$ and $\overline{B} = (4/5/6)$. We determine $\overline{A} \cdot \overline{B}$ using α-cuts and interval arithmetic. We compute $\overline{A}[\alpha] = [-3+\alpha, -1-\alpha]$ and $\overline{B}[\alpha] = [4+\alpha, 6-\alpha]$. So, if $\overline{C} = \overline{A} \cdot \overline{B}$ we obtain $\overline{C}[\alpha] = [(\alpha-3)(6-\alpha), (-1-\alpha)(4+\alpha)]$, $0 \leq \alpha \leq 1$. The graph of $\overline{C}$ is shown in Figure 2.3.

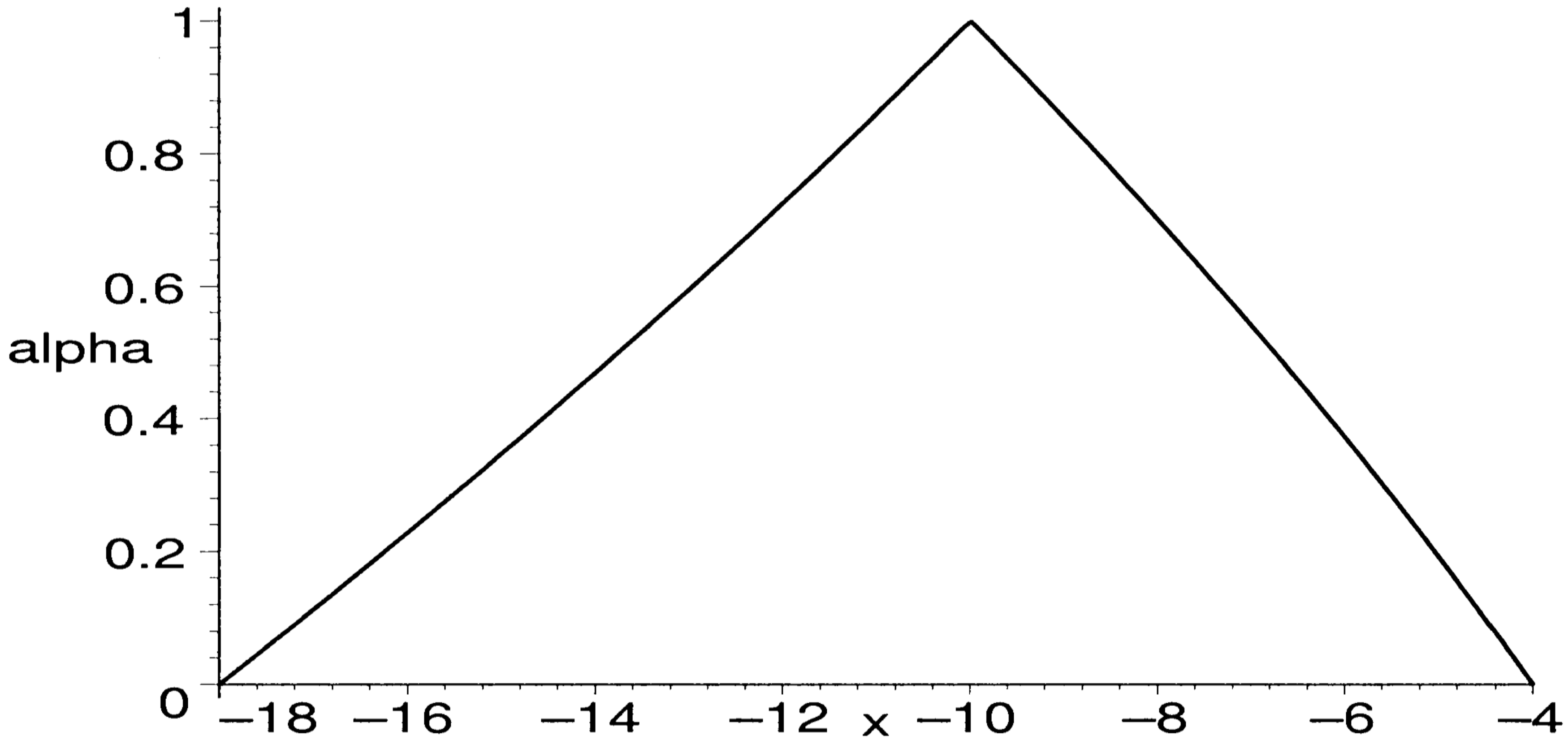

Figure 2.3: The Fuzzy Number $\overline{C} = \overline{A} \cdot \overline{B}$