Fuzzy Statistics

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Estimate μ, Variance Known

Chapter 4

4.1 Fuzzy Estimator of μ

Consider X a random variable with probability density function $N(\mu, \sigma^2)$, which is the normal probability density with unknown mean μ and unknown variance σ^2. To estimate μ we obtain a random sample $X_1, X_2, \ldots,$ X_n from $N(\mu, \sigma^2)$.

Suppose the mean of this random sample turns out to be $\bar{x}$, which is a crisp number, not a fuzzy number.

4.1 Fuzzy Estimator of μ

Also, let S^2 be the sample variance. Our point estimator of μ, is $\bar{x}$. If the values of the random sample are $X_1, X_2, \ldots, X_n$ then the expression we will use for S^2 is:

$$s^2 = \sum_{i=1}^{n} (x_i - \overline{x})^2/(n-1)$$

We will use this form of S^2 , with denominator (n -1), so that it is an unbiased estimator of σ^2.

4.1 Fuzzy Estimator of μ

It is known that $\frac{(\bar{x} - \mu)}{(s/\sqrt{n})}$ has a (Student's) t distribution with (n – 1) degrees of freedom. It follows that.

$$P(-t_{\beta/2} \leq \frac{\overline{x} - \mu}{s/\sqrt{n}} \leq t_{\beta/2}) = 1 - \beta$$

where $t_{\beta/2}$ is defined from the (Student's) t distribution, with n - 1 degrees of freedom, so that the probability of exceeding it is $\beta/2$. Now solve the inequality for μ, giving

4.1 Fuzzy Estimator of μ

Now solve the inequality for μ, giving

$$P(\overline{x} - t_{\beta/2}s/\sqrt{n} \leq \mu \leq \overline{x} + t_{\beta/2}s/\sqrt{n}) = 1 - \beta.$$

For this we immediately obtain the $(1-\beta)\%100$ confidence interval for μ,

$$[\overline{x} - t_{\beta/2}s/\sqrt{n}, \overline{x} + t_{\beta/2}s/\sqrt{n}].$$

Put these confidence intervals together, we obtain fl our fuzzy number estimator of μ.

Example 4.1.1

Consider X a random variable with probability density function $N(\mu,\sigma^2)$, which is the normal probability density with unknown mean μ and unknown variance. To estimate μ we obtain a random sample $X_1, X_2,\ldots, X_n$ from N(μ,σ^2). Suppose the mean of this random sample of size 25 turns out to be 28.6 and S^2=3.42 . Then a $(1-\beta)\%100$ confidence interval for μ is

$$[28.6 - t_{\beta/2}\sqrt{3.42/25}, 28.6 + t_{\beta/2}\sqrt{3.42/25}]$$

To obtain a graph of fuzzy μ, or $\bar{\mu}$, first assume that $0.01 \leq \beta \leq 1$. We will use MATLAB to create the Graph of function.

Example 4.1.1

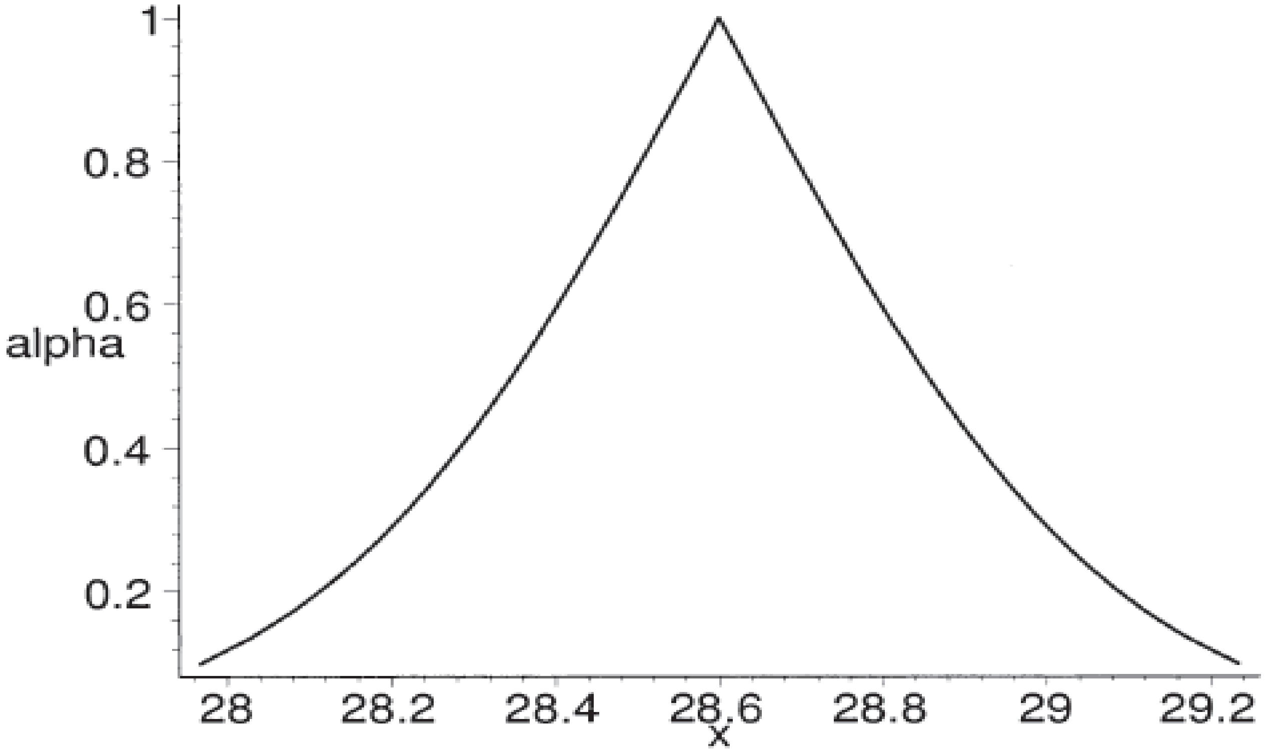

Figure 4.2: Fuzzy Estimator $\overline{\mu}$ in Example 4.1.1, $0.10 \leq \beta \leq 1$

Example 4.1.1

```
>> x=linspace(26,30);
>> y=linspace(0.01,1);
>> f1=28.6-0.3699*icdf('T',(1-y/2),24);
>> f2=28.6+ 0.3699 *icdf('T',(1-y/2),24);
>> plot(f1,y,f2,y)
>> ylabel ('alpha')
>> xlabel('mean')
```

Example 4.1.1