Fuzzy Statistics

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Estimate p, Binomial Population

Chapter 5

5.1 Fuzzy Estimator of *p*

We have an experiment in mind in which we are interested in only two possible outcomes labeled "success" and "failure". Let ***p*** be the probability of a success so that $q = 1 - p$ will be the probability of a failure. We want to estimate the value of ***p***. let we have a random sample which here is running the experiment n independent times and counting the number of times we had a success.

5.1 Fuzzy Estimator of *p*

Let x be the number of times we observed a success in n independent repetitions of this experiment. Then our point estimate of p is: $\hat{p}=\frac{x}{n}$

We know that $\frac{(\hat{p}-\mathrm{p})}{\sqrt{p(1-\mathrm{p})/n}}$ is approximately $N(0,1)$ if n is sufficiently large. Then

$$P(z_{\beta/2} \leq \frac{\widehat{p}-p}{\sqrt{p(1-p)/n}} \leq z_{\beta/2}) \approx 1-\beta$$

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where $z\_(\beta/2)$ is defined as:

$$\int_{-\infty}^{z_{\beta/2}} N(0,1)dx = 1 - \beta/2,$$

Solving the inequality for the ***p*** in the numerator we have $P(\widehat{p} - z_{\beta/2}\sqrt{p(1-p)/n} \leq p \leq \widehat{p} + z_{\beta/2}\sqrt{p(1-p)/n}) \approx 1 - \beta.$

This leads directly to the $(1-\beta)$%100 confidence interval for ***p.*** $[\widehat{p} - z_{\beta/2}\sqrt{p(1-p)/n}, \widehat{p} + z_{\beta/2}\sqrt{p(1-p)/n}]$

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However, we have no value for p to use in this confidence interval. So, still assuming that n is sufficiently large, we substitute $\hat{p}$ for p in the equation using $\hat{q} = 1 - \hat{p}$, and we get the final (1−β)%100 approximate confidence interval

$$[\widehat{p} - z_{\beta/2}\sqrt{\widehat{p}\widehat{q}/n}, \widehat{p} + z_{\beta/2}\sqrt{\widehat{p}\widehat{q}/n}]$$

Put these confidence intervals together we get $\hat{p}$ our triangular shaped fuzzy number estimator of *p*.

Example 5.1.1

Assume that n = 350, x = 180 so that *p* = 0.5143. The confidence intervals become.

$$[0.5143 - 0.0267 z_{\beta/2}, 0.5143 + 0.0267 z_{\beta/2}]$$

To obtain a graph of fuzzy *p*, or $\bar{p}$, first assume that $0.01 \leq \beta \leq 1$.

Example 5.1.1

```
>> x=linspace(0,1);
>> y=linspace(0.1,1);
>> f1=0.5143-0.0267*icdf('Normal',(1-y/2));
>> f2=0.5143+0.0267*icdf('Normal',(1-y/2));
>> plot(f1,y,f2,y)
>> ylabel ('alpha')
>> xlabel('x')
```

Example 5.1.1

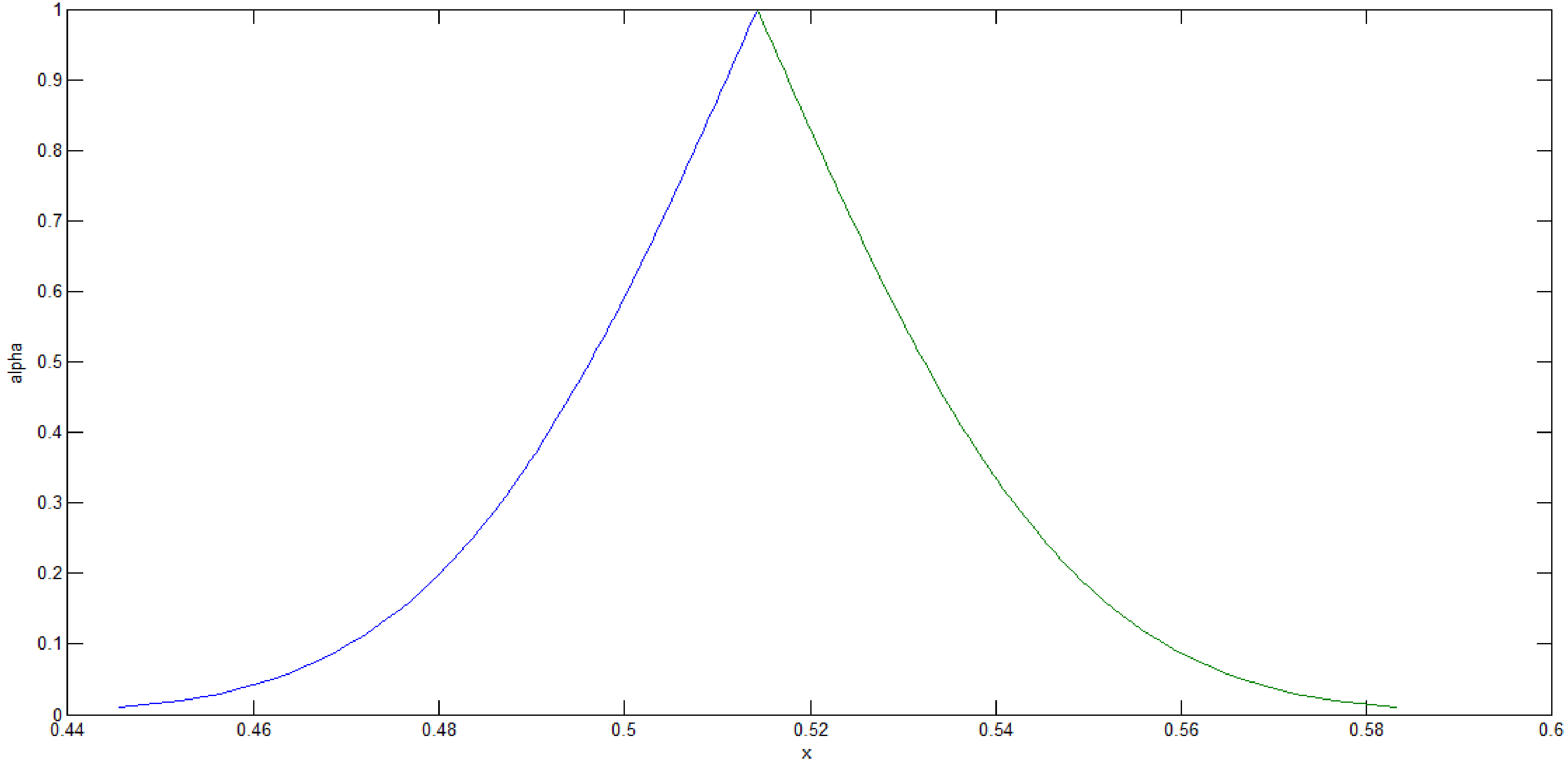

Figure 5.3: Fuzzy Estimator $\overline{p}$ in Example 5.1.1, $0.01 \leq \beta \leq 1$

Example 5.1.1

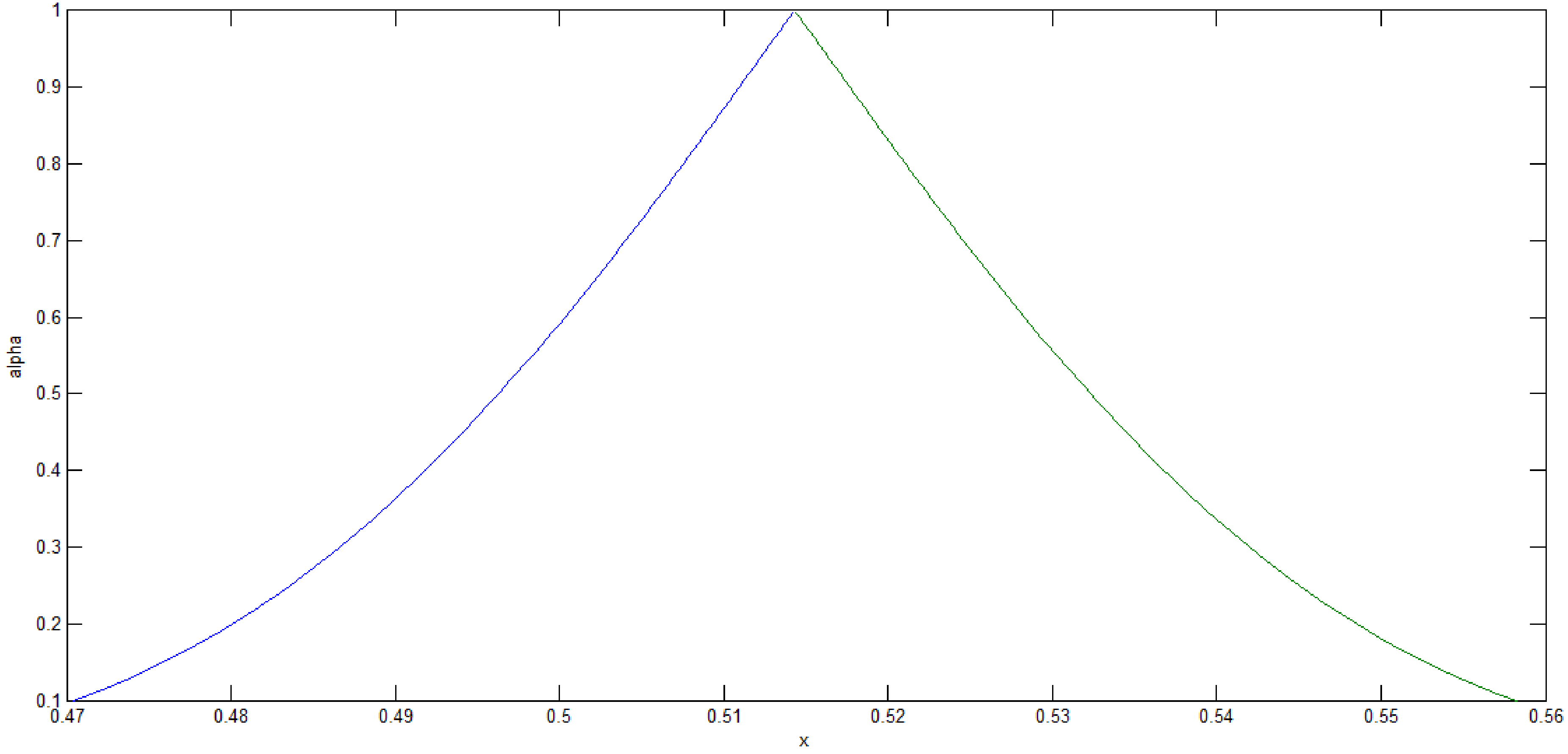

Figure 5.2: Fuzzy Estimator $\overline{p}$ in Example 5.1.1, $0.10 \leq \beta \leq 1$