Fuzzy Statistics

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Estimate σ^2 *from a Normal Population*

Chapter 6

6.2 Biased Fuzzy Estimator

Consider X a random variable with probability density function $N(\mu, \sigma^2)$, which is the normal probability density with unknown mean μ and unknown variance σ^2. To estimate σ^2 we obtain a random sample $X_1, X_2, \ldots, X_n$ from $N(\mu, \sigma^2)$.

Our point estimator for the variance will be S^2 • If the values of the random sample are $x_1, x_2, \ldots x_n$ then the expression we will use for S^2 is

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$$s^2 = \sum_{i=1}^{n} (x_i - \overline{x})^2 / (n-1)$$

We will use this form of S^2 , with denominator (n -1), so that it is an unbiased estimator of σ^2.
We know that $(n-1)S^2/\sigma^2$has a chi-square distribution with n - 1 degrees of freedom. Then

$$P(\chi^2_{L,\beta/2} \leq (n-1)s^2/\sigma^2 \leq \chi^2_{R,\beta/2}) = 1 - \beta,$$

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where $\chi^2_{R,\beta/2}$ $(\chi^2_{L,\beta/2})$ is the point on the right (left) side of the density where χ^2 the probability of exceeding (being less than) it is $\beta/2$. The χ^2 distribution has n - 1 degrees of freedom. Solve the inequality for σ^2 and we see that

$$P(\frac{(n-1)s^2}{\chi^2_{R,\beta/2}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{L,\beta/2}}) = 1 - \beta.$$

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From this we obtain the usual (1−β)%100 confidence intervals for σ^2

$$[(n-1)s^2/\chi^2_{R,\beta/2}, (n-1)s^2/\chi^2_{L,\beta/2}]$$

Put these confidence intervals together, as discussed in Chapter 3, and we obtain $\bar{\sigma}^2$ our fuzzy number estimator of σ^2.

We now show that this fuzzy estimator is biased because the vertex of the triangular shaped fuzzy number $\bar{\sigma}^2$,

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where the membership value equals one, is not at S^2 • We say a fuzzy estimator is biased when its vertex is not at the point estimator. We obtain the vertex of $\bar{\sigma}^2$ when (β = 1.0) Let

$$factor = \frac{n-1}{\chi^2_{R,0.50}} = \frac{n-1}{\chi^2_{L,0.50}},$$

after we substitute β = 1 Then the 0% confidence interval for the variance $[(factor)(s^2), (factor)(s^2)] = (factor)(s^2)$

Since $factor \neq 1$ the fuzzy number $\bar{\sigma}^2$ is not centered at S^2.

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n	$factor$
10	1.0788
20	1.0361
50	1.0138
100	1.0068
500	1.0013
1000	1.0007

Table 6.1: Values of $factor$ for Various Values of n

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Table 1 shows some values of factor for various choices for n. We see that $factor \to 1 \; as \; n \to \infty$ but factor is substantially larger than one for small values on n. This fuzzy estimator is biased.

6.3 Unbiased Fuzzy Estimator

In deriving the usual confidence interval for the variance we start with recognizing that $(n-1)S^2/\sigma^2$ has a x^2 distribution with n -1 degrees of freedom. Then for a (1−β)%100 confidence interval we may find a and b so that

$$\mathrm{p}\left(\mathrm{a} \leq \frac{(n-1)S^2}{\sigma^2} \leq b\right) = 1-\beta$$

The usual confidence interval has a and b so that the probabilities in the "two tails" are equal.

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That is, $a = \chi^2_{L,\beta/2}$ $(b = \chi^2_{R,\beta/2})$ that the probability of being less (greater) than a (b) is $\beta/2$.

Assume that $0.01 \leq \beta \leq 1$. Now this interval for β is fixed and also n and S^2 are fixed. Define

$$L(\lambda) = [1 - \lambda]\chi^2_{R,0.005} + \lambda(n - 1),$$

$$R(\lambda) = [1 - \lambda]\chi^2_{L,0.005} + \lambda(n - 1)$$

Then a confidence interval for the variance is

$$[\frac{(n-1)s^2}{L(\lambda)}, \frac{(n-1)s^2}{R(\lambda)}]$$

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For $0 \leq \lambda \leq 1$, start with a 99% confidence interval when $\gamma = 0$ and end up with a 0% confidence interval for $\gamma = 1$

Our confidence interval for σ, the population standard deviation, is

$$[\sqrt{(n-1)/L(\lambda)}s, \sqrt{(n-1)/R(\lambda)}s]$$

Example 6.3.1

Consider X a random variable with probability density function $N(\mu, \sigma^2)$, which is the normal probability density with unknown mean μ and unknown variance σ^2. To estimate σ^2 we obtain a random sample $X_1, X_2, \ldots, X_n$ from N(μ,σ^2). the random sample of size 25 and S^2 =3.42.Then a $(1-\beta)\%100$ confidence interval for σ^2 is $[\frac{82.08}{L(\lambda)}, \frac{82.08}{R(\lambda)}]$.

To obtain a graph of fuzzy σ^2, or $\bar{\sigma}^2$, first assume that $0.01 \leq \beta \leq 1$. We will use MATLAB to create the Graph of function.

Example 6.3.1

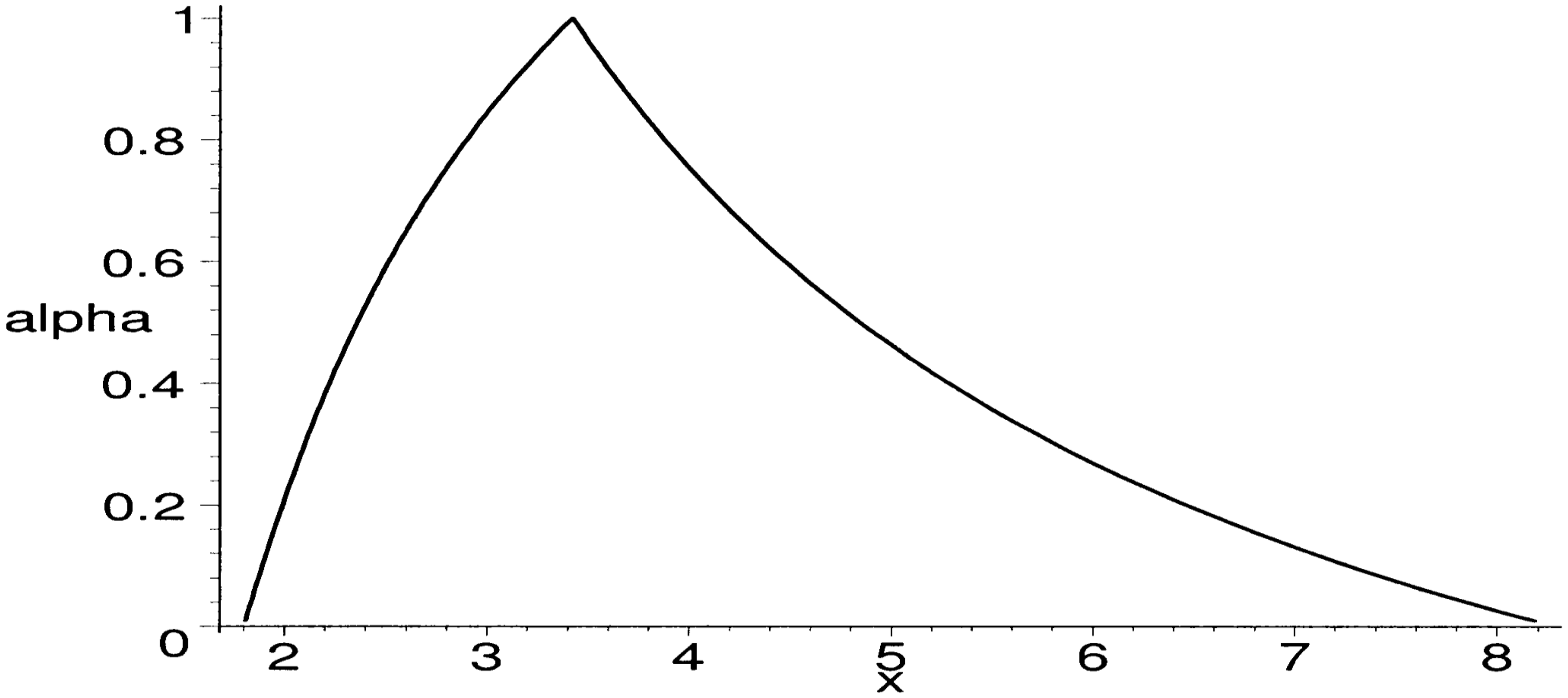
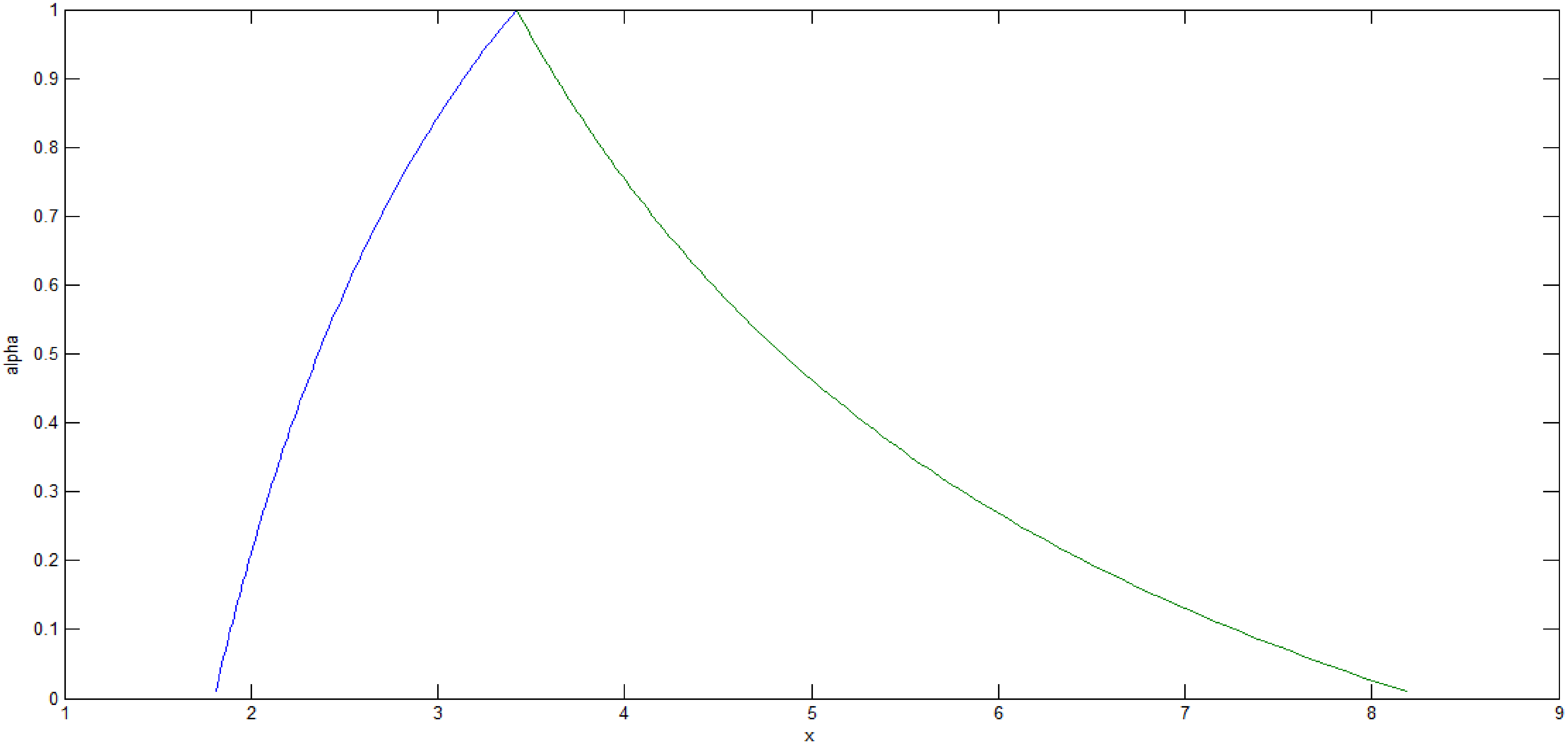


Figure 6.1: Fuzzy Estimator $\overline{\sigma}^2$ in Example 6.3.1, $0.01 \leq \beta \leq 1$

Example 6.3.1



Example 3.3.1

```
x=linspace(0,1);
y=linspace(0.01,1);
X2L= chi2inv(.995,24);
X2R= chi2inv(.005,24);
f1=(1-y)* X2L + y*24;
f2=(1-y)* X2R + y*24;
f11=82.08./f1;
f22=82.08./f2;
plot(f11,y,f22,y)
ylabel ('alpha')
xlabel('x')
```