# Fuzzy Statistics

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# Estimate $\mu_{1-}\ \mu_2$

Chapter 7

# 7.1 Estimate μ1- μ2 ,Variance Known

We have two populations: Pop I and Pop II. Pop I is normally distributed with unknown mean $\mu_1$ and known variance $\sigma_1^2$. Pop II is also normally distributed with unknown mean $\mu_2$ but known variance $\sigma_1^2$. We wish to construct a fuzzy estimator for $\mu_1 - \mu_2$. We collect a random sample of size $n_1$ from Pop I and let $\bar{x}_1$ be the mean for this data. We also gather a random sample of size $n_2$ from Pop II and $\bar{x}_2$ is the mean for the second sample. We assume these two random samples are independent.

# 7.1 Estimate μ1- μ2 ,Variance Known

Now $\bar{x}_1 - \bar{x}_2$ is normally distributed with mean $\mu_1 - \mu_2$ and standard deviation $\sigma_0 = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ Then as in Section 3.3 of Chapter 3, $(1-\beta)\%100$ confidence interval for $\mu_1 - \mu_2$, is

$$\overline{x}_1 - \overline{x}_2 - z_{\beta/2}\sigma_0, \overline{x}_1 - \overline{x}_2 + z_{\beta/2}\sigma_0$$

we place these confidence intervals one on top of another to build our fuzzy estimator $\bar{\mu}_{12}$ for $\mu_1 - \mu_2$

# Example 7.1.1

Assume that: (1) $n_1 = 15$, $\overline{x}_1 = 70.1$ ,$\sigma_1^2 = 6$; and (2) $n_2 = 8$, $\overline{x}_2 = 75.3$ ,$\sigma_2^2 = 4$. Then equation (7.1) becomes

$$[-5.2 - 0.9487 z_{\beta/2}, -5.2 + 0.9487 z_{\beta/2}]$$

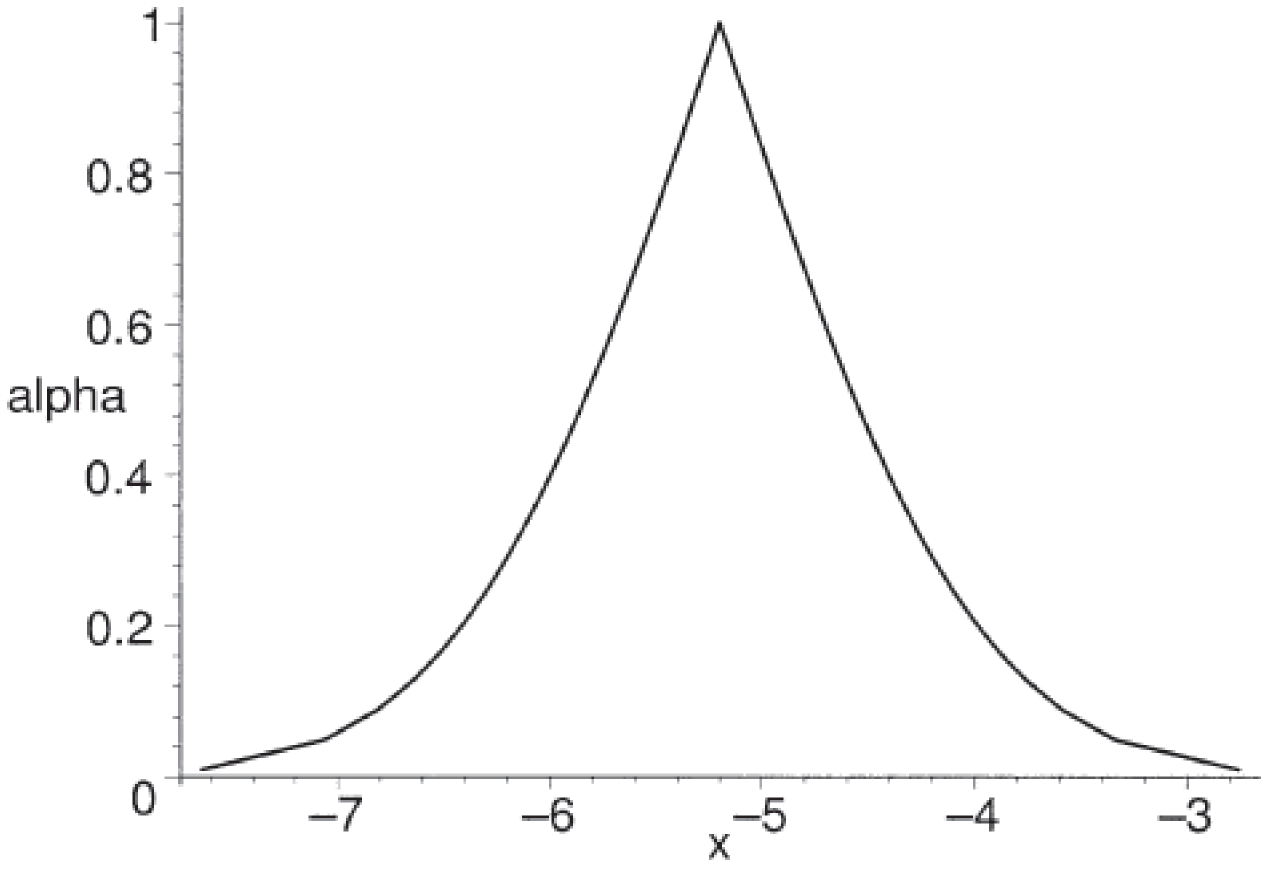


Figure 7.1: Fuzzy Estimator $\overline{\mu}_{12}$ in Example 7.1.1, $0.01 \leq \beta \leq 1$

# 8.1 Estimate μ1- μ2 , Variances Unknown: Large Samples

We assume that $n_1 > 30, n_2 > 30$. Let $S_1^2$, $S_1^2$. be the sample variance calculated from the data acquired from Pop I (Pop II). With large samples we may use the normal approximation and a $(1-\beta)\%100$ confidence interval for $\mu_1 - \mu_2$ is (Section 7.3 in [1]).

$$\overline{x}_1 - \overline{x}_2 - z_{\beta/2}s_0, \overline{x}_1 - \overline{x}_2 + z_{\beta/2}s_0]$$

Where $s_0 = \sqrt{s_1^2/n_1 + s_2^2/n_2}$. Put these confidence intervals together to obtain a fuzzy estimator $\bar{\mu}_{12}$ for the difference of the means. The results are similar to those in Chapter 7.

# 8.2 Estimate μ1- μ2 , Variances Unknown: Small Samples

Here we have two cases: (1) if we may assume that the variances are equal; or (2) the variances are not equal. We assume that $n_1 < 30, n_2 < 30$. Let $S_1^2$, $S_1^2$.

# 8.3.1 Estimate μ1- μ2 , Variances Unknown: Small Samples: Equal Variances

We have $\sigma_1^2 = \sigma_2^2 = \sigma^2$ • Define $S_p$ , the pooled estimator of the common variance, as,

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

Let $s_* = s_p\sqrt{1/n_1 + 1/n_2}$. Then it is known that

$$T = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s^*}$$

# 8.3.1 Estimate μ1- μ2 , Variances Unknown: Small Samples: Equal Variances

has a (Student's) t-distribution with $n_1 + n_2 - 2$ degrees of freedom. Then

$$P(-t_{\beta/2} \le T \le t_{\beta/2}) = 1 - \beta.$$

Solve the inequality for $\mu_1 - \mu_2$ and we find the $(1-\beta)\%100$ confidence interval for $\mu_1 - \mu_2$.

$$[\overline{x}_1 - \overline{x}_2 - t_{\beta/2}s^{*}, \overline{x}_1 - \overline{x}_2 + t_{\beta/2}s^{*}].$$

We place these confidence intervals one on top of another, as in Chapter 3, to get our fuzzy estimator $\bar{\mu}_{12}$.

# Example 8.3.1.1

Assume that the variances in the two populations are equal. Let the data be:(1) $n_1 = 15, \bar{x}_1 = 70.1, s_1 = 6$ $and$ $n_2 = 8, \bar{x}_2 = 75.3, s_2 =$ We compute $S_p = 2.3094$. Then $[-5.2 - 1.0110 t_{\beta/2}, -5.2 + 1.0110 t_{\beta/2}]$.

The degrees of freedom is 21. if $0.01 \le \beta \le 1$,then the graph of the fuzzy estimator $\bar{\mu}_{12}$ ,is in Figure 8.1.

# Example 8.3.1.1

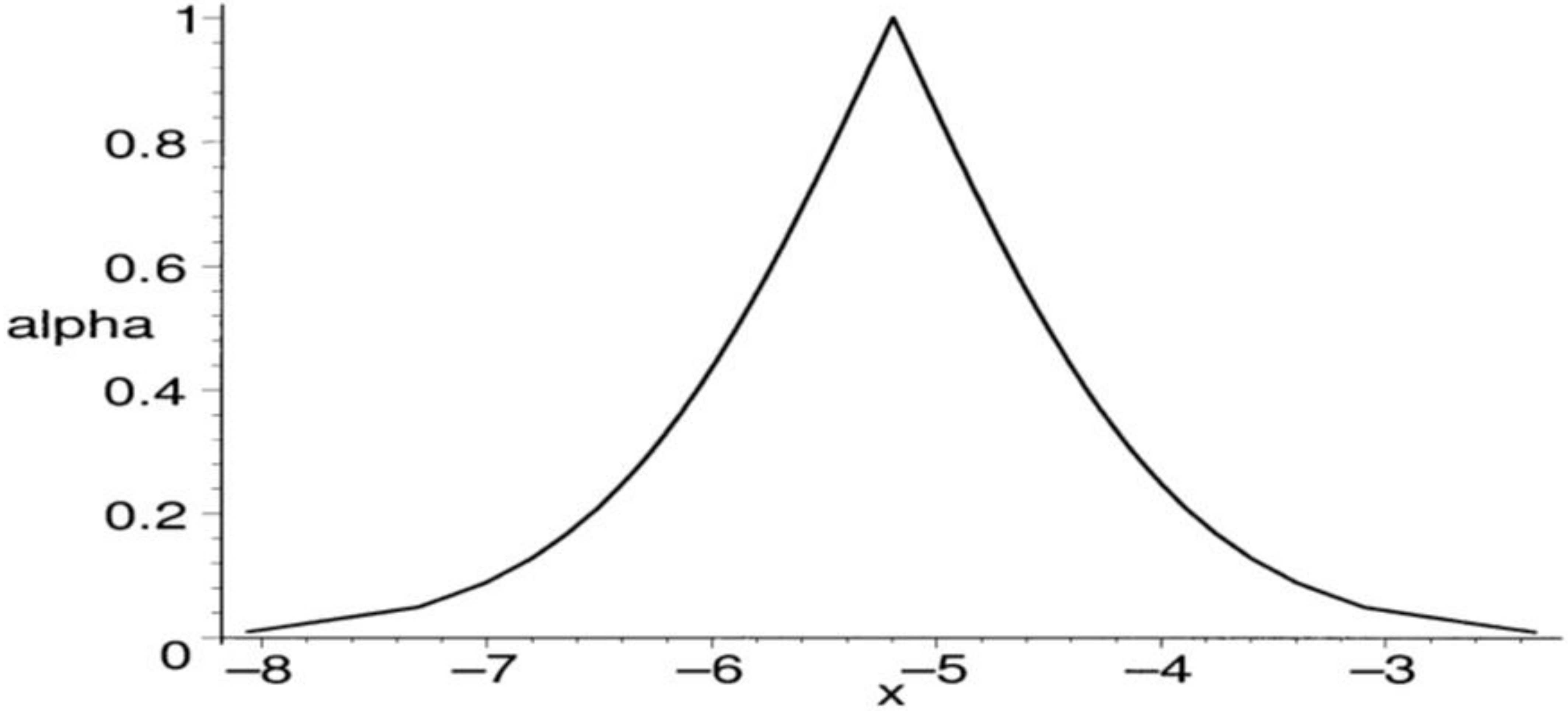


Figure 8.1: Fuzzy Estimator $\overline{\mu}_{12}$ in Example 8.3.1.1, $0.01 \leq \beta \leq 1$

# 8.3.2 Estimate μ1- μ2 , Variances Unknown: Small Samples: Unequal Variances

We have $\sigma_1^2 \neq \sigma_2^2$. Define $S_p$ , It is known that

$$T = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{s_0}.$$

is approximately (Student's) t distributed with r degrees of freedom. We find the degrees of freedom (r) by rounding up to the nearest integer the following expression

# 8.3.2 Estimate μ1- μ2 , Variances Unknown: Small Samples: Unequal Variances

$$\frac{(A+B)^2}{\frac{A^2}{n_1-1}+\frac{B^2}{n_2-1}},$$

Where $\mathrm{A} = s_1^2/n_1$ and $\mathrm{B} = s_2^2/n_2$. Let

$$T = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{s_0}.$$

Then

$$P(-t_{\beta/2} \leq T \leq t_{\beta/2}) \approx 1 - \beta.$$

Solve the inequality for $\mu_1 - \mu_2$ and we find the $(1-\beta)\%100$ confidence interval for $\mu_1 - \mu_2$.

# 8.3.2 Estimate μ1- μ2 , Variances Unknown: Small Samples: Unequal Variances

$$[\overline{x}_1 - \overline{x}_2 - t_{\beta/2}s_0, \overline{x}_1 - \overline{x}_2 + t_{\beta/2}s_0].$$

The t distribution has r degrees of freedom. We place these confidence intervals one on top of another, as in Chapter 3, to get our fuzzy estimator $\bar{\mu}_{12}$.

# Example 8.3.2.1

Let us use the same data as in Example 8.3.1.1 except now we do not assume the variances are ,the data be:(1) $n_1 = 15, \bar{x}_1 = 70.1, s_1 = 6 \; and \; n_2 = 8, \bar{x}_2 = 75.3, s_2 =$ We compute $S_p = 2.3094$.

An approximate $(1-\beta)\%100$ confidence interval for the difference of the means is

$$[-5.2 - (0.9487)t_{\beta/2}, -5.2 + (0.9487)t_{\beta/2}],$$

# Example 8.3.2.1

The term $S_o$ was defined in Section 8.2 and we computed it as 0.90487. Also , we determined that the degrees of freedom is r = 18.

if_0.01≤$\beta$≤1,then the graph of the fuzzy estimator $\mu$¯_12 ,is in Figure 8.2.

# Example 8.3.2.1

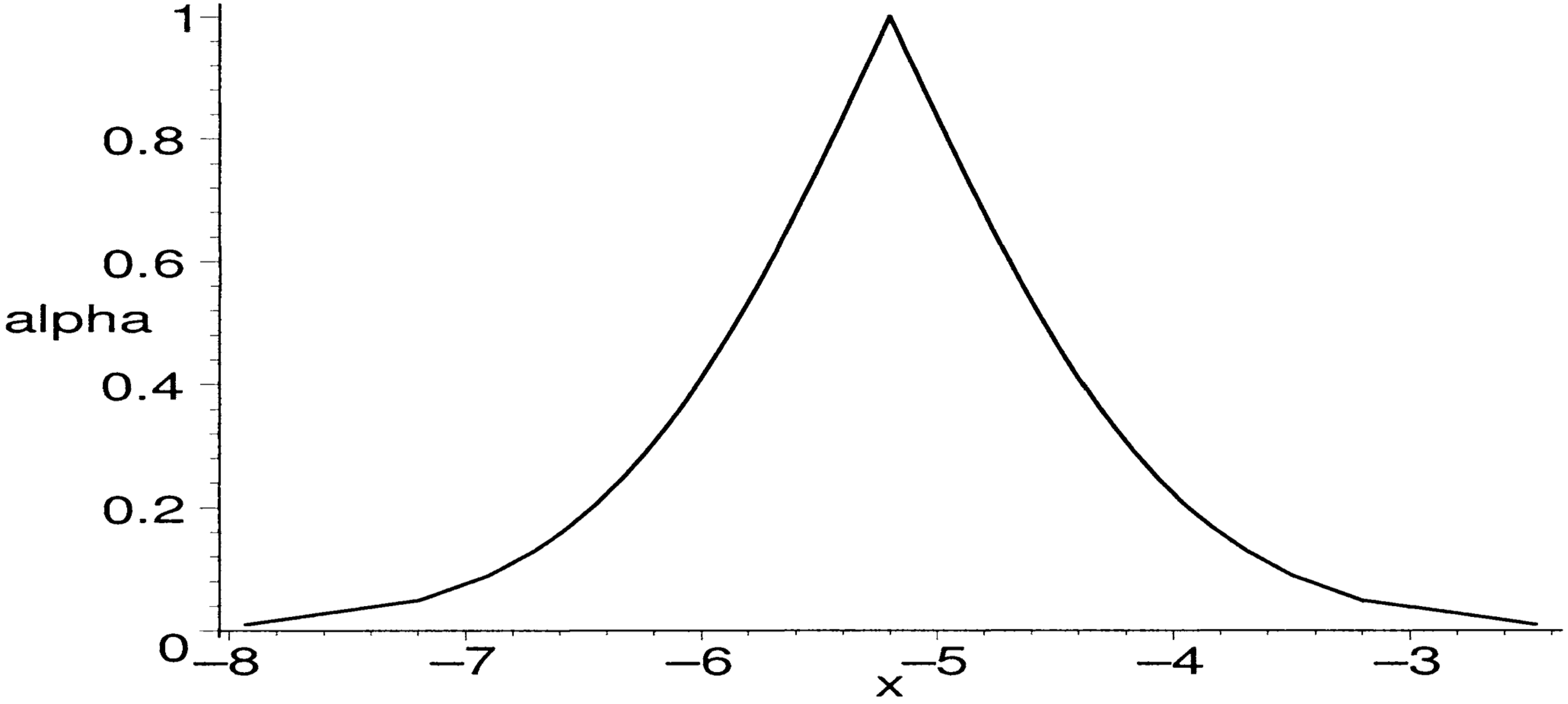


Figure 8.2: Fuzzy Estimator $\overline{\mu}_{12}$ in Example 8.3.2.1, $0.01 \leq \beta \leq 1$

# 9.1 Estimate d=μ1- μ2 , Matched Pairs

Let $x_1, x_2,\ldots, x_n$ be the values of a random sample from a population Pop 1. Let *object*$_i$ or person$_i$, belong to Pop I which produced measurement $x_i$ in the random sample, $1 < i < n$. Then, at possibly some later time, we take a second measurement on *object*$_i$ (person$_i$)and get value $y_i$,$1 < i < n$. Then ( $(x_1, y_1), \cdots (x_n, y_n)$ ,are n pairs of dependent measurements.

# 9.1 Estimate d=μ1- μ2 , Matched Pairs

For example, when testing the effectiveness of some treatment for high blood pressure, the $x_i$ are the blood pressure measurements before treatment and the $y_i$ are these measurements on the same person after treatment. The two samples are not independent so we can not use the results of Chapters 7 or 8.

# 9.1 Estimate d=µ1- µ2 , Matched Pairs

Let $d_i = x_i - y_i$ , $1 < i < n$. Next compute the mean $\bar{d}$ (crisp number , not fuzzy) and the variance $S_d^2$ of the $d_i$ data. Assume that n > 30 so we may use the normal approximation; or assume that the $d_i$ are approximately normally distributed with unknown mean $\mu_d$ and unknown variance $\sigma_d^2$. Then

$$T = \frac{d - \mu_d}{s_d / \sqrt{n}},$$

# 9.1 Estimate d=μ1- μ2 , Matched Pairs

has a t distribution with n - 1 degrees of freedom. It follows that: $P(-t_{\beta/2} \leq T \leq t_{\beta/2}) = 1 - \beta.$

From this it immediately follows that a $(1-\beta)\%100$ confidence interval for $\mu_d$ is

$$[\overline{d} - t_{\beta/2}\frac{s_d}{\sqrt{n}}, \overline{d} + t_{\beta/2}\frac{s_d}{\sqrt{n}}].$$

Now place these confidence intervals given in above equation one on top of another to produce our fuzzy estimator $\bar{\mu}_d$ of $\mu_d$.

# Example 9.1.1

Consider the paired data in the below Table. This table contains a weeks forecast high temperatures and the actual recorded high values.

| Forecast ($x$) | Actual High ($y$) |
|---|---|
| 68 | 72 |
| 76 | 74 |
| 66 | 62 |
| 72 | 76 |
| 76 | 75 |
| 80 | 78 |
| 71 | 75 |

# Example 9.1.1

We compute $d_i = x_i - y_i\,, 1 < i < 7$. and then $\bar{d} = -0.4286, S_d = 3.4572$.

The (1−$\beta$)%100 confidence interval are

$$[-0.4286 - (1.3067)t_{\beta/2}, -0.4286 + (1.3067)t_{\beta/2}].$$

We graphed the confidence intervals for 0.01≤$\beta$≤1, and the result is our fuzzy estimator $\bar{\mu}_d$ of $\mu_d$ in Figure 9.1.

# Example 9.1.1

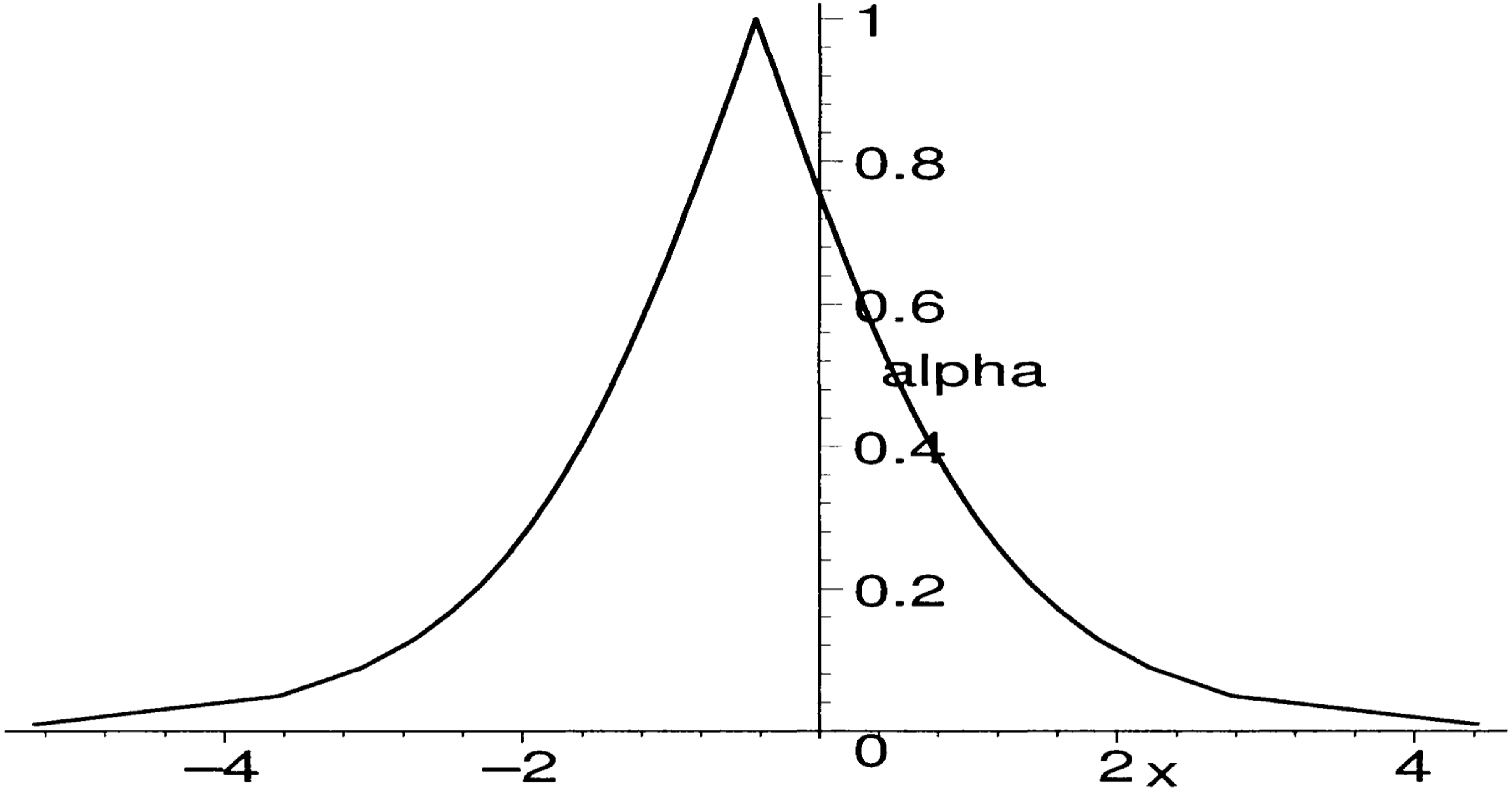


Figure 9.1: Fuzzy Estimator $\overline{\mu}_d$ in Example 9.1.1, $0.01 \leq \beta \leq 1$