أدارة المشاريع

منهج كمي

البرمجة الخطية في ادارة المشاريع

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❖ استخدام البرمجة الخطية في ايجاد المسار الحرج لشبكة المشروع

$$\text{Maximize } Z = \sum_{i=1}^{n} T_{N,i} . Y_i$$

حيث ان :

$T_{N,i}$: الوقت الطبيعي للنشاط i ويكون عادة الوقت لأكمال النشاط باقل الموارد

Y_i: متغير القرار لوقت البداية للنشاط i

- دالة الهدف هذه تخضع الى القيود التالية :

1- للانشطة التي تدخل الى العقدة i $\sum_{i=1}^{n} Y_i = 1$

2- للانشطة التي تخرج من العقدة i $\sum_{i=1}^{n} -Y_i = -1$

3- للانشطة التي تدخل و تخرج من العقدة i:

لكل عقدة، هناك عقبة واحدة تمثل الحفاظ على التدفق:

اجمالي التدفق الداخل=اجمالي التدفق الخارج

**في هذه الصيغة، $Y_i = 0 \text{ or } 1$ تدل على غياب أو وجود تدفق وحدة من عقدة إلى أخرى.لذلك المعادلة تكون:

$$\sum_{i=1}^{n} +Y - Y_i = 0$$

4- قيود عدم السالبية:

لجميع متغيرات القرار $Y_i \geq 0$

- استخدام البرمجة الخطية في ايجاد الوقت المرغوب به لانجاز المشروع.

و تتضمن الاتي:

- تكلفة التعجيل المرتبطة بوحدة الوقت لجميع الأنشطة:

للحد من الوقت لاستكمال النشاط، يتم تطبيق المزيد من الموارد في شكل أفراد إضافيين والعمل الإضافي، وتقصير مدة انجاز النشاط، ولكن التكاليف ترتفع.

عندما يتم تطبيق أقصى جهد يمكن أن يكتمل النشاط في أقصر وقت ممكن .

اتكلفة الميل تظهر مقدار تكلفة العمل ستتغير اذا كانت اسرعت الأنشطة أو تباطأت.

- معرفة شبكة المشروع مع وقت كل نشاط و التي يمكن ايجادها بإسلوب CPM .
- الى أي مدى يمكن ان يتم ضغط النشاط.
  - المتغيرات لهذه المشكلة:

Y_i: وقت وقوع الحدث i، يتم قياسه من بداية المشروع، حيث ان i=(1,2,3,……,n)

X_q: مجموع اوقات الانشطة التي سيتم تعجيلها (تقاس بالايام، الاسابيع،الاشهر)، حيث ان: q=(1,2,3,….,n)

U_q: كلفة الميل او كلفة تعجيل وحدة واحدة من الزمن لكل نشاط q.

- دالة الهدف تكون:

$$\text{Minimize } Z = \sum_{q=1}^{L} U_q \,.\, X_q$$

- تخضع الى الشروط التالية:

1- قيود الوقت المعجل:

$X_q \geq$ وقت الضغط المسموح به للنشاط q

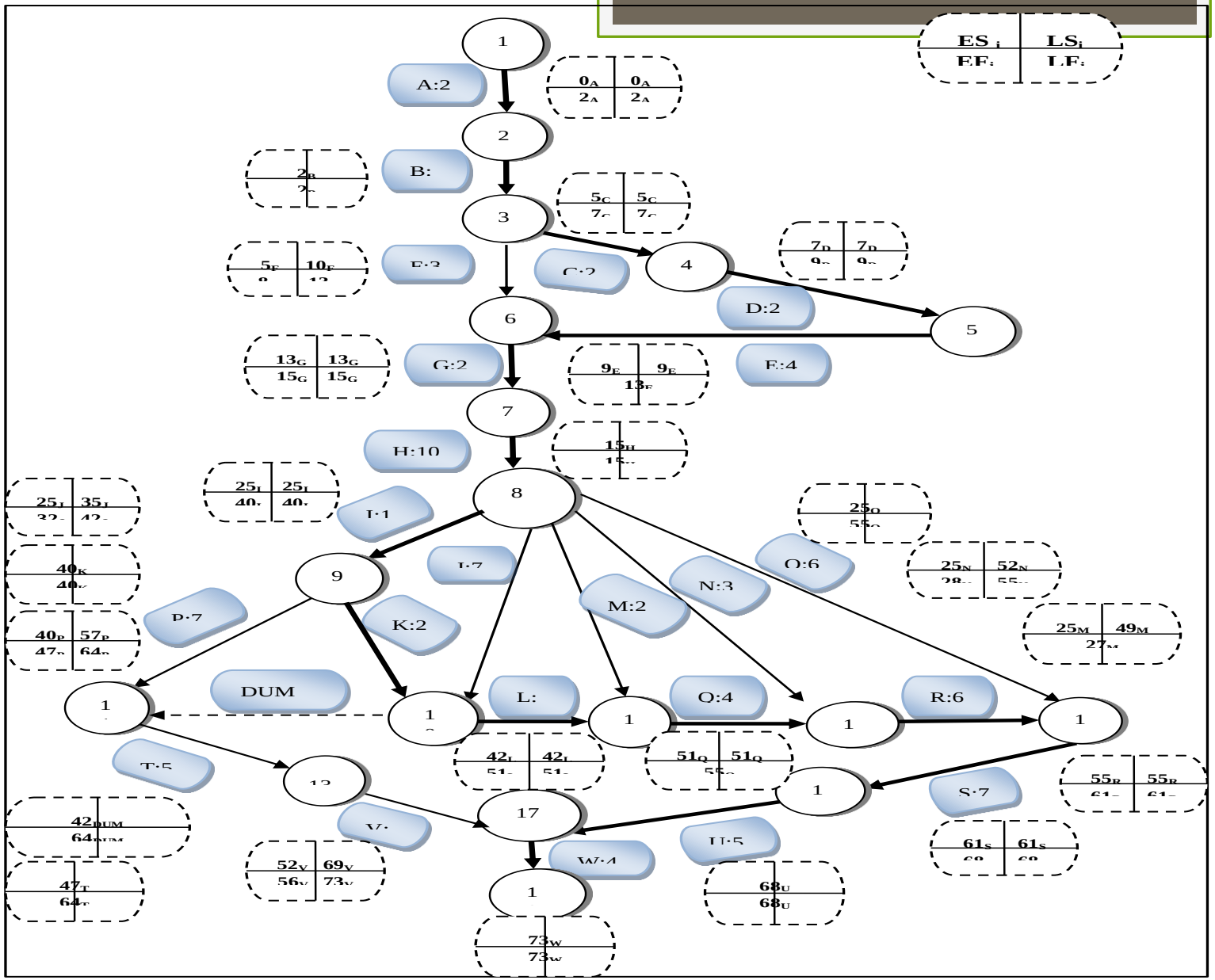
2- القيود التي تكشف الشبكة:

$(Yi) \leq$ (وقت البدء للنشاط + الوقت الطبيعي للنشاط – الوقت المعجل للنشاط)

3- قيود اكمال المشروع:

$Y_m \geq$ الموعد النهائي للمشروع بعد الضغط ، حيث m تشير إلى الحدث الأخير من هذا المشروع

4- قيود عدم السالبية: جميع متغيرات القرار يجب ان تكون اكبر او تساوي الصفر.


ES i | LS i
EF i | LF i
1
A:2
2
B:
3
C:2
4
D:2
5
E:3
6
F:4
G:2
7
H:10
8
I:1
9
K:2
L:
M:2
N:3
O:6
P:7
Q:4
R:6
S:7
T:5
DUM
17
U:5
W:4

- مثال على استخدام البرمجة الخطية في ايجاد المسار الحرج لشبكة المشروع

$$MAXIMIZE\ Z = 2A + 3B + 2C\ + 2D + 4E + 3F + 2G + 10H + 15\ I + 7\ J +\ 2M + 3N + 6O + 7P + 2K + 9L + 4Q + 6R + 0\ DUM + 5T + 4V + 7S + 5U +\ \ \ 4W$$

- **Subject to:**

For activities that leaves node (i)

node 1 $-A = -1$

For activities that entered and leaves node (i):

node 2 $A - B = 0$

node 3 $B - C - F = 0$

node 4 $C - D = 0$

node 5 $D - E = 0$

node 6 $F + E - G = 0$

node 7 $G - H = 0$

node 8 $H - I - J - M - N - O = 0$

node 9 $I - P - K = 0$

node 10 $J + K - L - DUM = 0$

node 11 $P + DUM - T = 0$

node 12 $M + L - Q = 0$

node 13 $T - V = 0$

node 14 $N + Q - R = 0$

node 15 $O + R - S = 0$

node 16 $S - U = 0$

node 17 $V + U - W = 0$

For activities that entered node (i)

node 18 $W = 1$

- Nonnegative constraints.

All the decision variable $Y_i \geq 0$ i.e. $A, B, \ldots, W \geq 0$

- يتم ايجاد حل المشكلة بواسطة برنامج WIN Q.S.B و كالتالي:

No. of Variable	Decision variable	Solution value	Unit cost or profit c(j)	Total contribution	Reduced cost	Basis status	Allowable min. c(j)	Allowable max. c(j)
1	A	1	2	2	0	basic	–M	M
2	B	1	3	3	0	basic	–M	M
3	C	1	2	2	0	basic	–3	M
4	D	1	2	2	0	basic	–3	M
5	E	1	4	4	0	basic	–1	M
6	F	0	3	0	–5	at bound	–M	8
7	G	1	2	2	0	basic	–M	M
8	H	1	10	10	0	basic	–M	M
9	I	1	15	15	0	basic	5	M
10	J	0	7	0	–10	at bound	–M	17
11	M	0	2	0	–24	at bound	–M	26
12	N	0	3	0	–27	at bound	–M	30
13	O	0	6	0	–30	at bound	–M	36
14	P	0	7	0	–17	at bound	–M	24
15	K	1	2	2	0	basic	–8	M
16	L	1	9	9	0	basic	–8	M
17	Q	1	4	4	0	basic	–13	M
18	R	1	6	6	0	basic	–11	M
19	A_{dum}	0	0	0	–22	at bound	–M	22
20	T	0	5	0	0	basic	–M	22
21	V	0	4	0	0	basic	–M	21
22	S	1	7	7	0	basic	–10	M
23	U	1	5	5	0	basic	–12	M
24	W	1	4	4	0	basic	–M	M
Objective Function (Max.) = 77								

- مثال على استخدام البرمجة الخطية في ايجاد وقت انجاز المشروع في الوقت المرغوب به.

سيتم اعتماد نفس المثال السابق

Activities	Tail event (starting event)	Head event (ending event)
A	1	2
B	2	3
C	3	4
D	4	5
E	5	6
F	3	6
G	6	7
H	7	8
I	8	9
J	8	10
M	8	12
N	8	14

Activities	Tail event (starting event)	Head event (ending event)
O	8	15
P	9	11
K	9	10
L	10	12
Q	12	14
R	14	15
DUM	10	11
T	11	13
V	13	17
S	15	16
U	16	17
W	17	18

Let, Y_1: Time when event 1 will occur.

Y_2: Time when event 2 will occur.

Y_3: Time when event 3 will occur.

Y_4: Time when event 4 will occur.

Y_5: Time when event 5 will occur.

Y_6: Time when event 6 will occur.

Y_7: Time when event 7 will occur.

Y_8: Time when event 8 will occur.

Y_9: Time when event 9 will occur.

Y_{10}: Time when event 10 will occur.

Y_{11}: Time when event 11 will occur.

Y_{12}: Time when event 12 will occur.

Y_{13}: Time when event 13 will occur.

Y_{14}: Time when event 14 will occur.

Y_{15}: Time when event 15 will occur.

Y_{16}: Time when event 16 will occur.

Y_{17}: Time when event 17 will occur.

Y_{18}: Time when event 18 will occur

X_A: Number of weeks activity A will be crashed.

X_B: Number of weeks activity B will be crashed.

X_C: Number of weeks activity C will be crashed.

X_D: Number of weeks activity D will be crashed.

X_E: Number of weeks activity E will be crashed

X_F: Number of weeks activity F will be crashed.

X_G: Number of weeks activity G will be crashed.

X_H: Number of weeks activity H will be crashed.

X_I: Number of weeks activity I will be crashed.

X_J: Number of weeks activity J will be crashed

X_M: Number of weeks activity M will be crashed.

X_N: Number of weeks activity N will be crashed.

X_O: Number of weeks activity O will be crashed.

X_P: Number of weeks activity P will be crashed.

X_K: Number of weeks activity K will be crashed

X_L: Number of weeks activity L will be crashed.

X_Q: Number of weeks activity Q will be crashed.

X_{DUM}: Number of weeks activity DUM will be crashed.

X_T: Number of weeks activity T will be crashed.

X_V: Number of weeks activity V will be crashed

X_S: Number of weeks activity S will be crashed.

X_U: Number of weeks activity U will be crashed

- دالة الهدف تكون:

$$MINIMIZE\ (Z) = 10000X_A + 20000X_B + 10000X_C + 10000X_D + 25000X_E + 30000X_F + 100000X_G + 40000X_H + 40000X_I + 25000X_J + 5000X_K + 40000X_L + 5000X_M + 10000X_N + 15000X_O + 40000X_P + 75000X_Q + 70000X_S + 50000X_T + 35000X_U + 35000X_V + 25000X_W + 0X_{DUM}$$

- تخضع الى القيود التالية:

1) Maximum reduction constraints

$X_A \leq 1$

$X_B \leq 2$

$X_C \leq 1$

$X_D \leq 1$

$X_E \leq 2$

$X_F \leq 1$

$X_G \leq 1$

$X_H \leq 3$

$X_I \leq 5$

$X_J \leq 2$

$X_K \leq 1$

$X_{DUM} \leq 0$

$X_L \leq 3$

$X_M \leq 1$

$X_N \leq 1$

$X_O \leq 2$

$X_P \leq 3$

$X_Q \leq 2$

$X_S \leq 2$

$X_T \leq 2$

$X_U \leq 2$

$X_V \leq 2$

$X_W \leq 2$

2) Start time constraints:

$$Y_1 = 0$$

$$Y_1 + 2 - X_A \leq Y_2$$

$$Y_2 + 3 - X_B \leq Y_3$$

$$Y_3 + 2 - X_C \leq Y_4$$

$$Y_4 + 2 - X_D \leq Y_5$$

$$Y_5 + 4 - X_E \leq Y_6$$

or, $Y_3 + 3 - X_F \leq Y_6$

$$Y_6 + 2 - X_G \leq Y_7$$

$$Y_7 + 10 - X_H \leq Y_8$$

$$Y_8 + 15 - X_I \leq Y_9$$

$$Y_8 + 7 - X_J \leq Y_{10}$$

Or, $Y_9 + 2 - X_K \leq Y_{10}$

$$Y_8 + 2 - X_M \leq Y_{12}$$

Or, $Y_{10} + 9 - X_L \leq Y_{12}$

$$Y_8 + 3 - X_N \leq Y_{14}$$

Or, $Y_{12} + 4 - X_Q \leq Y_{14}$

$$Y_8 + 6 - X_O \leq Y_{15}$$

Or, $Y_{14} + 6 - X_R \leq Y_{15}$

$$Y_9 + 7 - X_P \leq Y_{11}$$

Or, $Y_{10} + 0 - X_{DUM} \leq Y_1$

$$Y_{11} + 5 - X_T \leq Y_{13}$$

$$Y_{15} + 7 - X_S \leq Y_{16}$$

$$Y_{16} + 5 - X_U \leq Y_{17}$$

Or, $Y_{13} + 4 - X_V \leq Y_{17}$

$$Y_{17} + 4 - X_W \leq Y_{18}$$

3) Project completion desire time

$Y_{18} \leq 50$

4) Nonnegative constraints.

All the decision variables $Y_i \geq 0$

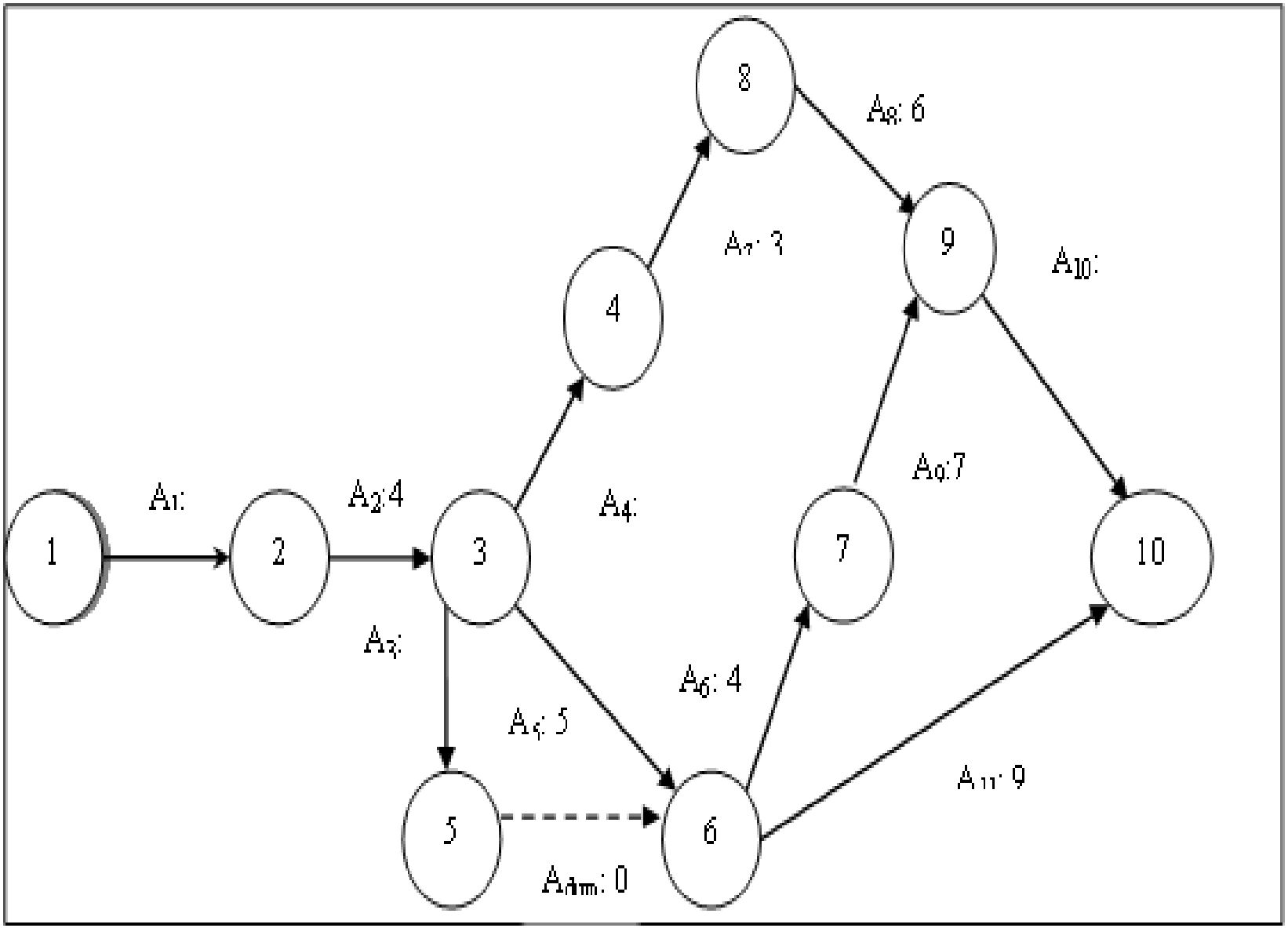
- يتم ايجاد حل المشكلة بواسطة برنامج WIN Q.S.B و كالتالي:

No. of variable	Decision variable	Solution value	Unit cost or profit c(j)	Total contribution	Reduced cost	Basis status	Allowable min. c(j)	Allowable max. c(j)
1	X_A	1	10,000	10,000	0	basic	–M	75,000
2	X_B	2	20,000	40,000	0	basic	–M	75,000
3	X_C	1	10,000	10,000	0	basic	–M	75,000
4	X_D	1	10,000	10,000	0	basic	–M	75,000
5	X_E	2	25,000	50,000	0	basic	–M	75,000
6	X_F	0	30,000	0	30,000	at bound	0	M
7	X_G	0	100,000	0	25,000	at bound	75,000	M
8	X_H	3	40,000	120,000	0	basic	–M	75,000
9	X_I	5	40,000	200,000	0	basic	–M	75,000
10	X_J	0	25,000	0	25,000	at bound	0	M
11	X_K	1	5,000	5,000	0	basic	–M	75,000
12	X_L	3	40,000	120,000	0	basic	–M	75,000
13	X_M	0	5,000	0.	5,000	at bound	0	M
14	X_N	0	10,000	0	10,000	at bound	0	M
15	X_O	0	15,000	0.0	15,000	at bound	0	M
16	X_P	0	40,000	0.00	40,000	at bound	0	M
17	X_Q	1	75,000	75,000	0	basic	70,000	75,000
18	X_R	3	70,000	210,000	0	basic	-M	75,000
19	X_S	0	75,000	0	0	at bound	75,000	M
20	X_T	0	50,000	0	50,000	at bound	0	M
21	X_U	2	35,000.	70,000	0	basic	–M	75,000
22	X_V	0	35,000	0	35,000	at bound	0	M
23	X_W	2	25,000	50,000	0	basic	–M	75,000
24	X_{DUM}	0	0	0	0	at bound	0	M
25	Y_1	0	0	0	0	basic	–M	M
26	Y_2	1	0	0	0	basic	–65,000	M
27	Y_3	2	0	0	0	basic	–55,000	M
28	Y_4	3	0	0	0	basic	–55,000	M
29	Y_5	4	0	0	0	basic	–55,000	M
30	Y_6	6	0	0	0	basic	–50,000	M
31	Y_7	8	0	0	0	basic	–50,000	25,000
32	Y_8	15	0	0	0	basic	–35,000	25,000
33	Y_9	25	0	0	0	basic	–35,000	25,000
34	Y_{10}	26	0	0	0	basic	–35,000	25,000
35	Y_{11}	39	0	0	0	basic	–35,000	0
36	Y_{12}	32	0	0	0	basic	–35,000	25,000
37	Y_{13}	44	0	0	0	basic	–35,000	0

الامثلة غير المحلولة

❑ مثال 1:

Activities code	Activity predecessor	Normal time T_n	Crash time T_c	Normal cost C_n	Crash cost C_c	Max reduction in time	Cost slope
A_1	---	3	2	5,000	7,000	1	2,000
A_2	A_1	4	2	4,000	5,000	2	500
A_3	A_2	4	4	7,000	7,000	0	---
A_4	A_2	3	1	3,000	5,000	2	1,000
A_5	A_2	5	2	6,000	10,500	3	1,500
A_6	A_5 , A_3	4	3	8,000	10,000	1	2,000
A_7	A_4	3	1	4,000	5,500	2	750
A_8	A_7	6	4	6,000	9,000	2	1,500
A_9	A_6	7	4	5,000	8,000	3	1,000
A_{10}	A_8, A_9	4	2	6,000	7,500	2	750
A_{11}	A_3, A_5	9	7	3,000	4,000	2	500
Total cost in normal and crash conditions				$57,000	$78,500		


1
2
3
4
5
6
7
8
9
10
A1:
A2:4
A3:
A4:
A5: 5
A6: 4
A7: 3
A8: 6
A9:7
A10:
A11: 9
Anim: 0

- استخدام البرمجة الخطية في ايجاد المسار الحرج:
- Maximize Z=

$3A_1+4A_2+4A_3+3A_4+5A_5+4A_6+3A_7+6A_8+7A_9+4$

$A_{10}+9A_{11}$

S.to:

1)For activities that leaves node (i):

Node 1 $-A_1= -1$

2) For activities that entered and leaves node (i):

Node 2 $A_1 - A_2 = 0$

Node 3 $A_2 - A_3 - A_4 - A_5 = 0$

Node 4 $A_4 - A_7 = 0$

Node 5 $A_3 - A_{Dum} = 0$

Node 6 $A_{Dum} + A_5 - A_6 - A_{11} = 0$

Node 7 $A_6 - A_7 = 0$

Node 8 $A_7 - A_8 = 0$

Node 9 $A_8 + A_9 - A_{10} = 0$

3) For activities that entered node (i):

Node 10 $A_{10} + A_{11} = 1$

4) Nonnegative constraints.

All the decision variable $Y_i \geq 0$ i.e

$A_1, A_2, \ldots\ldots\ldots, A_{11} \geq 0$

- استخدام البرمجة الخطية في ايجاد وقت انجاز المشروع في الوقت المرغوب به.

Activities	Start Event	End Event	Max reduction in time	Normal time T_n
A_1	1	2	1	3
A_2	2	3	2	4
A_3	3	5	0	4
A_4	3	4	2	3
A_5	3	6	3	5
A_{Dum}	5	6	0	0
A_6	6	7	1	4
A_7	4	8	2	3
A_8	8	9	2	6
A_9	7	9	3	7
A_{10}	9	10	2	4
A_{11}	6	10	2	9

Let,

Y_1: Time when event 1 will occur.

Y_2: Time when event 2 will occur.

Y_3: Time when event 3 will occur.

Y_4: Time when event 4 will occur.

Y_5: Time when event 5 will occur.

Y_6: Time when event 6 will occur.

Y_7:Time when event 7 will occur.

Y_8:Time when event 8 will occur.

Y_9: Time when event 9 will occur.

Y_{10}: Time when event 10 will occur.

X_{Dum}: Number of weeks activity A_{Dum} will be crashed.

X_1: Number of weeks activity A_1 will be crashed.

X_2: Number of weeks activity A_2 will be crashed.

X_3: Number of weeks activity A_3 will be crashed.

X_4: Number of weeks activity A_4 will be crashed.

X_5: Number of weeks activity A_5 will be crashed.

X_6: Number of weeks activity A_6 will be crashed.

X_7: Number of weeks activity A_7 will be crashed.

X_8: Number of weeks activity A_8 will be crashed.

X_9: Number of weeks activity A_9 will be crashed.

X_{10}: Number of weeks activity A_{10} will be crashed.

X_{11}: Number of weeks activity A_{11} will be crashed.

- Objective function:

Minimize Z=2000A_1+500 A_2+0 A_3+1000 A_4+1500 A_5+2000 A_6+750 A_7+1500 A_8+1000 A_9+750 A_{10}+500 A_{11}+0 A_{Dum}

- S.To:

1) Maximum reduction constraints

$X_1 \leq 1$

$X_2 \leq 2$

$X_3 \leq 0$

$X_4 \leq 2$

$X_5 \leq 3$

$X_{Dum} \leq 0$

$X_6 \leq 1$

$X_7 \leq 2$

$X_8 \leq 2$

$X_9 \leq 3$

$X_{10} \leq 2$

$X_{11} \leq 2$

2) Start time constraints:

Y_1=0

Y_1+3- $X_1 \leq Y_2$

Y_2+4- $X_2 \leq Y_3$

Y_3+3- $X_4 \leq Y_4$

Y_3+5- $X_3 \leq Y_5$

Y_5+0- $X_{Dum} \leq Y_6$

Y_3+5- $X_5 \leq Y_6$

Y_6+4- $X_6 \leq Y_7$

Y_4+3- $X_7 \leq Y_8$

Y_8+6- $X_8 \leq Y_9$

Y_7+7- $X_9 \leq Y_9$

Y_9+4- $X_{10} \leq Y_{10}$

Y_6+9- $X_{11} \leq Y_{10}$

3) Project completion desire time

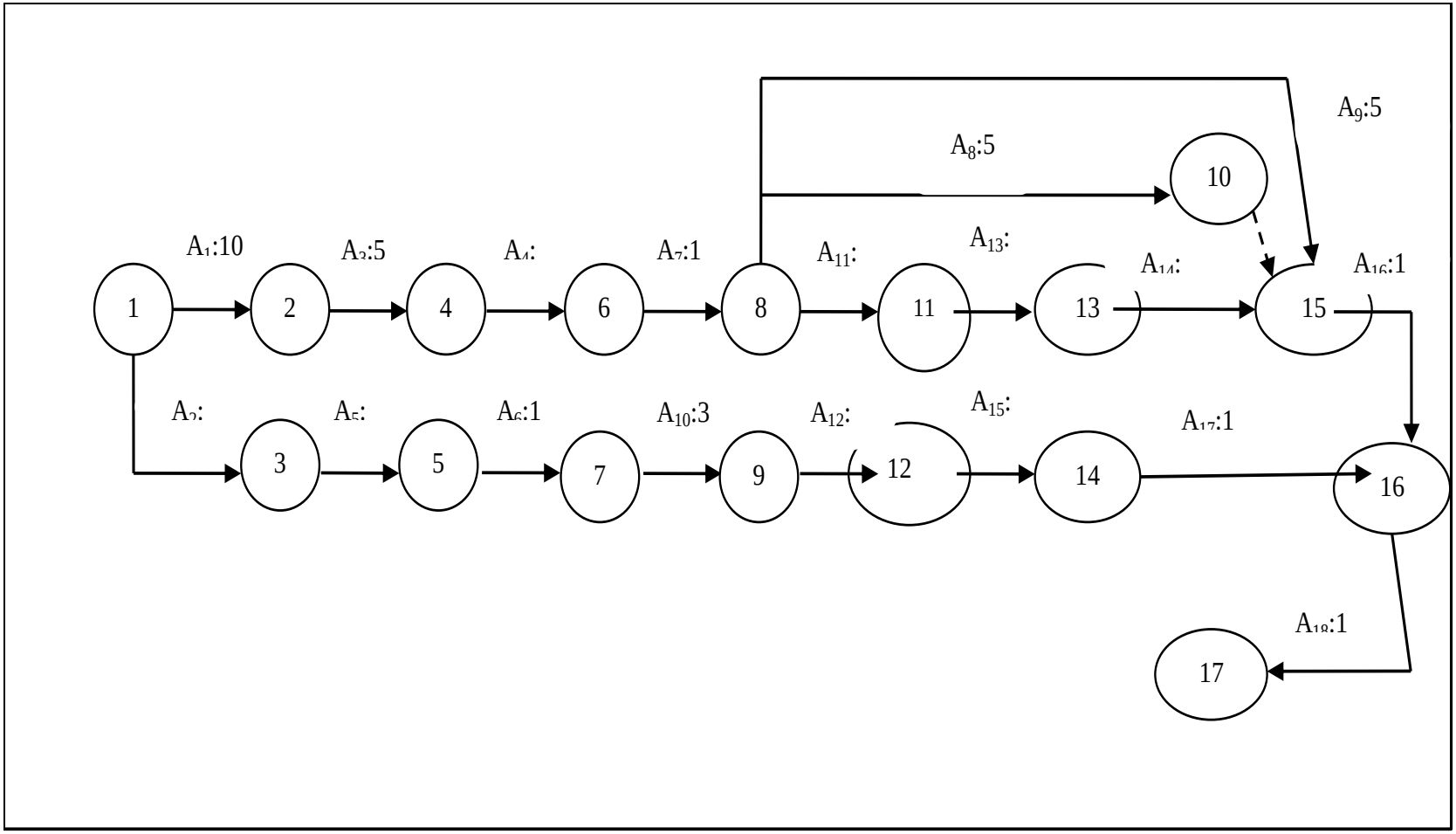
$Y_{11} \leq 15$

4) Nonnegative constraints.

All the decision variables $Y_i \geq 0$

مثال 2:

Activity code	Activity predecessor	Normal time T_n	Crash time T_c	Normal cost C_n	Crash cost C_c	Max reduction in time	Cost slope
A_1	---	10	8	20,000	28,000	2	4,000
A_2	---	3	2	8,000	10,000	1	2,000
A_3	A_1	5	3	10,000	15,000	2	2,500
A_4	A_3	5	3	8,500	12,000	2	1,750
A_5	A_2	4	3	4,000	5,000	1	1,000
A_6	A_5	10	8	14,000	19,000	2	2,500
A_7	A_4	10	8	14,000	16,000	2	1,000
A_8	A_7	5	3	5,000	6,000	2	500
A_9	A_7	5	3	9,000	11,000	2	1,000
A_{10}	A_6	3	3	2,500	2,500	0	0
A_{11}	A_7	3	3	2,500	2,500	0	0
A_{12}	A_{10}	5	4	13,000	15,000	1	2,000
A_{13}	A_{11}	5	3	12,000	15,000	2	1,500
A_{14}	A_{13}	5	3	10,000	12,000	2	1,000
A_{15}	A_{12}	5	2	8,000	14,000	3	2,000
A_{16}	A_8,A_9,A_{14}	10	7	15,000	18,000	3	1,000
A_{17}	A_{15}	10	6	13,000	19,000	4	1,500
A_{18}	A_{16},A_{17}	10	7	18,000	26,000	3	2,666.66
Total cost in normal and crash conditions				$186,500	$246,000		


A9:5
A8:5
10
A1:10
A2:5
A4:
A7:1
A11:
A13:
A14:
A16:1
1
2
4
6
8
11
13
15
A3:
A5:
A6:1
A10:3
A12:
A15:
A17:1
3
5
7
9
12
14
16
A18:1
17

- استخدام البرمجة الخطية في ايجاد المسار الحرج:

- Maximize Z= $10A_1$+3 A_2+5 A_3 +5 A_4+4 A_5+10 A_6+10 A_7+5 A_8+5 A_9+3 A_{10}+3 A_{11}+5 A_{12}+5 A_{13}+5 A_{14}+5 A_{15}+10 A_{16}+10 A_{17}+10 A_{18}

- S.to:

1)For activities that leaves node (i):

Node 1 $-A_1-A_2= -1$

2) For activities that entered and leaves node (i):

Node 2 $A_1 - A_3 = 0$

Node 3 $A_2 - A_5 = 0$

Node 4 $A_3 - A_4 = 0$

Node 5 $A_5 - A_6 = 0$

Node 6 $A_4 - A_7 = 0$

Node 7 $A_6 - A_{10} = 0$

Node 8 $A_7 - A_8 - A_9 - A_{11} = 0$

Node 9 $A_{10} - A_{12} = 0$

Node 10 $A_8 - A_{Dum} = 0$

Node 11 $A_{11} - A_{13} = 0$

Node 12 $A_{12} - A_{15} = 0$

Node 13 $A_{13} - A_{14} = 0$

Node 14 $A_{15} - A_{17} = 0$

Node 15 $A_9 + A_{Dum} + A_{14} - A_{16} = 0$

Node 16 $A_{16} + A_{17} - A_{18} = 0$

3) For activities that entered node (i):

Node 17 A_{18}=1

4) Nonnegative constraints.

All the decision variable $Y_i \geq 0$ i.e

$A_1, A_2, \ldots\ldots\ldots, A_{18} \geq 0$

- استخدام البرمجة الخطية في ايجاد وقت انجاز المشروع في الوقت المرغوب به.

Activities	Start Event	End Event	Max reduction in time	Normal time T_n
A_1	1	2	2	10
A_2	1	3	1	3
A_3	2	4	2	5
A_4	4	6	2	5
A_5	3	5	1	4
A_6	5	7	2	10
A_7	6	8	2	10
A_8	8	10	2	5
A_9	8	15	2	5

Activities	Start Event	End Event	Max reduction in time	Normal time T_n
A_{10}	7	9	0	3
A_{11}	8	11	0	3
A_{12}	9	12	1	5
A_{13}	11	13	2	5
A_{14}	13	15	2	5
A_{Dum}	10	15	0	0
A_{15}	12	14	3	5
A_{16}	15	16	3	10
A_{17}	14	16	4	10
A_{18}	16	17	3	10

Let, Y_1: Time when event 1 will occur.

Y_2: Time when event 2 will occur.

Y_3: Time when event 3 will occur.

Y_4: Time when event 4 will occur.

Y_5: Time when event 5 will occur.

Y_6: Time when event 6 will occur.

Y_7: Time when event 7 will occur.

Y_8: Time when event 8 will occur.

Y_9: Time when event 9 will occur.

Y_{10}: Time when event 10 will occur.

Y_{11}: Time when event 11 will occur.

Y_{12}: Time when event 12 will occur.

Y_{13}: Time when event 13 will occur.

Y_{14}: Time when event 14 will occur.

Y_{15}: Time when event 15 will occur.

Y_{16}: Time when event 16 will occur.

Y_{17}: Time when event 17 will occur.

- Let, X_{Dum}: Number of weeks activity A_{Dum} will be crashed.

X_1: Number of weeks activity A_1 will be crashed.

X_2: Number of weeks activity A_2 will be crashed.

X_3: Number of weeks activity A_3 will be crashed.

X_4: Number of weeks activity A_4 will be crashed.

X_5: Number of weeks activity A_5 will be crashed.

X_6: Number of weeks activity A_6 will be crashed.

X_7: Number of weeks activity A_7 will be crashed.

X_8: Number of weeks activity A_8 will be crashed.

X_9: Number of weeks activity A_9 will be crashed.

X_{10}: Number of weeks activity A_{10} will be crashed.

X_{11}: Number of weeks activity A_{11} will be crashed.

X_{12}: Number of weeks activity A_{12} will be crashed.

$\boldsymbol{X_{13}}$: Number of weeks activity $\boldsymbol{A_{13}}$ will be crashed.

$\boldsymbol{X_{14}}$: Number of weeks activity $\boldsymbol{A_{14}}$ will be crashed.

$\boldsymbol{X_{15}}$: Number of weeks activity $\boldsymbol{A_{15}}$ will be crashed.

$\boldsymbol{X_{16}}$: Number of weeks activity $\boldsymbol{A_{16}}$ will be crashed.

$\boldsymbol{X_{17}}$: Number of weeks activity $\boldsymbol{A_{17}}$ will be crashed.

$\boldsymbol{X_{18}}$: Number of weeks activity $\boldsymbol{A_{18}}$ will be crashed.

- Minimize Z=4000A_1+2000 A_2+2500 A_3+1750 A_4+1000 A_5+2500 A_6+1000 A_7+500 A_8+1000 A_9+0 A_{10}+0 A_{11}+2000 A_{12}+1500 A_{13}+1000 A_{14}+2000 A_{15}+1000 A_{16}+1500 A_{17}+2666.66 A_{18}+0 A_{Dum}
- S.to:

1) Maximum reduction constraints

$X_1 \leq 2$

$X_2 \leq 1$

$X_3 \leq 2$

$X_4 \leq 2$

$X_5 \leq 1$

$X_6 \leq 2$

$X_7 \leq 2$

$X_8 \leq 2$

$X_9 \leq 2$

$X_{10} \leq 0$

$X_{11} \leq 0$

$X_{12} \leq 1$

$X_{13} \leq 2$

$X_{14} \leq 2$

$X_{Dum} \leq 0$

$X_{15} \leq 3$

$X_{16} \leq 3$

$X_{17} \leq 4$

$X_{18} \leq 3$

2) Start time constraints:

Y_1=0

Y_1+10 -$X_1 \leq Y_2$

Y_1+3 -$X_2 \leq Y_3$

Y_2+5 -$X_3 \leq Y_4$

Y_3+4 -$X_5 \leq Y_5$

Y_4+5 -$X_4 \leq Y_6$

Y_5+1 -$X_6 \leq Y_7$

Y_6+1 -$X_7 \leq Y_8$

Y_7+3 -$X_{10} \leq Y_9$

Y_8+5 -$X_8 \leq Y_{10}$

Y_8+3 -$X_{11} \leq Y_{11}$

Y_9+5 -$X_{12} \leq Y_{12}$

Y_{11}+5 -$X_{13} \leq Y_{13}$

Y_{12}+5 -$X_{15} \leq Y_{14}$

Y_8+5 -$X_9 \leq Y_{15}$

Y_{10}+0 -$X_{Dum} \leq Y_{15}$

Y_{13}+5 -$X_{14} \leq Y_{15}$

Y_{15}+1 -$X_{16} \leq Y_{16}$

Y_{14}+1 -$X_{17} \leq Y_{16}$

Y_{16}+1 -$X_{18} \leq Y_{17}$

3) Project completion desire time

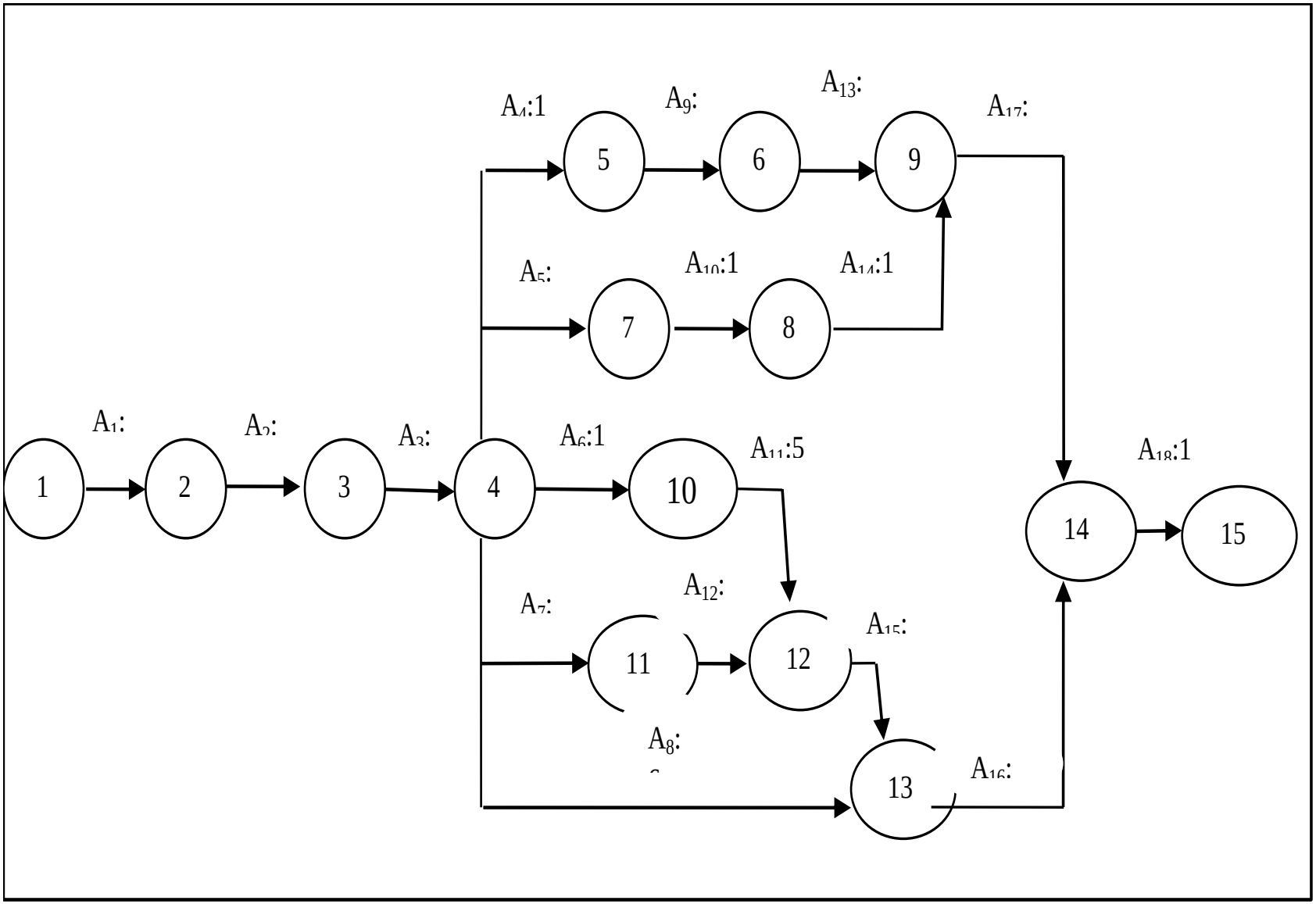
$Y_{17} \leq 55$

4) Nonnegative constraints.

All the decision variables $Y_i \geq 0$

❑ مثال 3:

Activity code	Activity predecessor	Normal time T_n	Crash time T_c	Normal cost C_n	Crash cost C_c	Max reduction in time	Cost slope
A_1	---	3	2	2,500	3,500	1	1,000
A_2	A_1	2	1	4,000	6,000	1	2,000
A_3	A_2	2	1	6,000	9,000	1	3,000
A_4	A_3	15	10	25,000	35,000	5	2,000
A_5	A_3	4	2	3,000	5,000	2	1,000
A_6	A_3	10	6	40,000	60,000	4	5,000
A_7	A_3	1	1	5,000	5,000	0	0
A_8	A_3	6	3	7,500	14,000	3	2,166.66
A_9	A_4	2	2	5,000	5,000	0	0
A_{10}	A_5	10	7	8,000	10,000	3	666.66
A_{11}	A_6	5	5	6,000	6,000	0	0
A_{12}	A_7	5	3	4,000	5,000	2	500
A_{13}	A_9	8	6	12,000	14,000	2	1,000
A_{14}	A_{10}	10	6	10,000	16,000	4	1,500
A_{15}	A_{11},A_{12}	3	3	6,500	6,500	0	0
A_{16}	A_8,A_{15}	5	4	7,000	8,000	1	1,000
A_{17}	A_{13},A_{14}	2	1	4,000	5,000	1	1,000
A_{18}	A_{16},A_{17}	1	1	3,000	3,000	0	0
Total cost in normal and crash conditions				$158,500	$216,000		


A1:
A2:
A3:
A4:1
A5:
A6:1
A7:
A8:
A9:
A10:1
A11:5
A12:
A13:
A14:1
A15:
A16:
A17:
A18:1
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

- استخدام البرمجة الخطية في ايجاد المسار الحرج:

- Maximize Z= 3 A_1+2 A_2+2 A_3+15 A_4+4 A_5+10 A_6+1 A_7+6 A_8+2 A_9+10 A_{10} +5 A_{11}+5 A_{12}+8 A_{13}+10 A_{14}+3 A_{15}+5 A_{16}+2 A_{17}+1 A_{18}+0 A_{Dum}

- S.to:

1)For activities that leaves node (i):

Node 1 $-A_1$= -1

2) For activities that entered and leaves node (i):

Node 2 $A_1 - A_2 = 0$

Node 3 $A_2 - A_3 = 0$

Node 4 $A_3 - A_4 - A_5 - A_6 - A_7 - A_8 = 0$

Node 5 $A_4 - A_9 = 0$

Node 6 $A_9 - A_{13} = 0$

Node 7 $A_5 - A_{10} = 0$

Node 8 $A_{10} - A_{14} = 0$

Node 9 $A_{13} + A_{14} - A_{17} = 0$

Node 10 $A_6 - A_{11} = 0$

Node 11 $A_7 - A_{12} = 0$

Node 12 $A_{11} + A_{12} - A_{15} = 0$

Node 13 $A_8 + A_{15} - A_{16} = 0$

Node 14 $A_{16} + A_{17} - A_{18} = 0$

3) For activities that entered node (i):

Node 15 A_{18}=1

4) Nonnegative constraints.

All the decision variable $Y_i \geq 0$ i.e

$A_1, A_2, \ldots\ldots\ldots, A_{18} \geq 0$

- استخدام البرمجة الخطية في ايجاد وقت انجاز المشروع في الوقت المرغوب به.

Activities	Start Event	End Event	Max reduction in time	Normal time T_n
A_1	1	2	1	3
A_2	2	3	1	2
A_3	3	4	1	2
A_4	4	5	5	15
A_5	4	7	2	4
A_6	4	10	4	10
A_7	4	11	0	1
A_8	4	13	3	6
A_9	5	6	0	2

Activities	Start Event	End Event	Max reduction in time	Normal time T_n
A_{10}	7	8	3	10
A_{11}	10	12	0	5
A_{12}	11	12	2	5
A_{13}	6	9	2	8
A_{14}	8	9	4	10
A_{15}	12	13	0	3
A_{16}	13	14	1	5
A_{17}	9	14	1	2
A_{18}	14	15	0	1

Let, Y_1: Time when event 1 will occur.

Y_2: Time when event 2 will occur.

Y_3: Time when event 3 will occur.

Y_4: Time when event 4 will occur.

Y_5: Time when event 5 will occur.

Y_6: Time when event 6 will occur.

Y_7: Time when event 7 will occur.

Y_8: Time when event 8 will occur.

Y_9: Time when event 9 will occur.

Y_{10}: Time when event 10 will occur.

Y_{11}: Time when event 11 will occur.

Y_{12}: Time when event 12 will occur.

Y_{13}: Time when event 13 will occur.

Y_{14}: Time when event 14 will occur.

Y_{15}: Time when event 15 will occur.

- Let, X_1: Number of weeks activity A_1 will be crashed.

X_2: Number of weeks activity A_2 will be crashed.

X_3: Number of weeks activity A_3 will be crashed.

X_4: Number of weeks activity A_4 will be crashed.

X_5: Number of weeks activity A_5 will be crashed.

X_6: Number of weeks activity A_6 will be crashed.

X_7: Number of weeks activity A_7 will be crashed.

X_8: Number of weeks activity A_8 will be crashed.

X_9: Number of weeks activity A_9 will be crashed.

X_{10}: Number of weeks activity A_{10} will be crashed.

X_{11}: Number of weeks activity A_{11} will be crashed.

X_{12}: Number of weeks activity A_{12} will be crashed.

$\boldsymbol{X_{13}}$: Number of weeks activity $\boldsymbol{A_{13}}$ will be crashed.

$\boldsymbol{X_{14}}$: Number of weeks activity $\boldsymbol{A_{14}}$ will be crashed.

$\boldsymbol{X_{15}}$: Number of weeks activity $\boldsymbol{A_{15}}$ will be crashed.

$\boldsymbol{X_{16}}$: Number of weeks activity $\boldsymbol{A_{16}}$ will be crashed.

$\boldsymbol{X_{17}}$: Number of weeks activity $\boldsymbol{A_{17}}$ will be crashed.

$\boldsymbol{X_{18}}$: Number of weeks activity $\boldsymbol{A_{18}}$ will be crashed

- Minimize Z= 1000 A_1+2000 A_2+3000 A_3+2000 A_4+1000 $A5$ +5000 A_6+0 A_7+2166.66 A_8+0 A_9+666.66 A_{10}+0 A_{11}+500 A_{12}+1000 A_{13}+1500 A_{14}+0 A_{15}+1000 A_{16}+1000 A_{17}+0 A_{18}
- S.To:

1) Maximum reduction constraints

$X_1 \leq 1$

$X_2 \leq 1$

$X_3 \leq 1$

$X_4 \leq 5$

$X_5 \leq 2$

$X_6 \leq 4$

$X_7 \leq 0$

$X_8 \leq 3$

$X_9 \leq 0$

$X_{10} \leq 3$

$X_{11} \leq 0$

$X_{12} \leq 2$

$X_{13} \leq 2$

$X_{14} \leq 4$

$X_{15} \leq 0$

$X_{16} \leq 1$

$X_{17} \leq 1$

$X_{18} \leq 0$

2) Start time constraints:

Y_1=0

Y_1+3-$X_1 \leq Y_2$

Y_2+2- $X_2 \leq Y_3$

Y_3+2- $X_3 \leq Y_4$

Y_4+1- $X_4 \leq Y_5$

Y_5+2- $X_9 \leq Y_6$

Y_4+4- $X_5 \leq Y_7$

Y_7+1- $X_{10} \leq Y_8$

Y_6+8- $X_{13} \leq Y_9$

Y_4+1- $X_6 \leq Y_{10}$

Y_4+1- $X_7 \leq Y_{11}$

Y_{10}+5- $X_{11} \leq Y_{12}$

Y_{11}+5- $X_{12} \leq Y_{12}$

Y_4+6- $X_8 \leq Y_{13}$

Y_{12}+8- $X_{13} \leq Y_{13}$

Y_9+2- $X_{17} \leq Y_{14}$

Y_{13}+5- $X_{16} \leq Y_{14}$

Y_{14}+1- $X_{18} \leq Y_{15}$

3) Project completion desire time

$Y_{15} \leq 24$

4) Nonnegative constraints.

All the decision variables $Y_i \geq 0$

Thank you